# In the laboratory of Mathematics

Triangles

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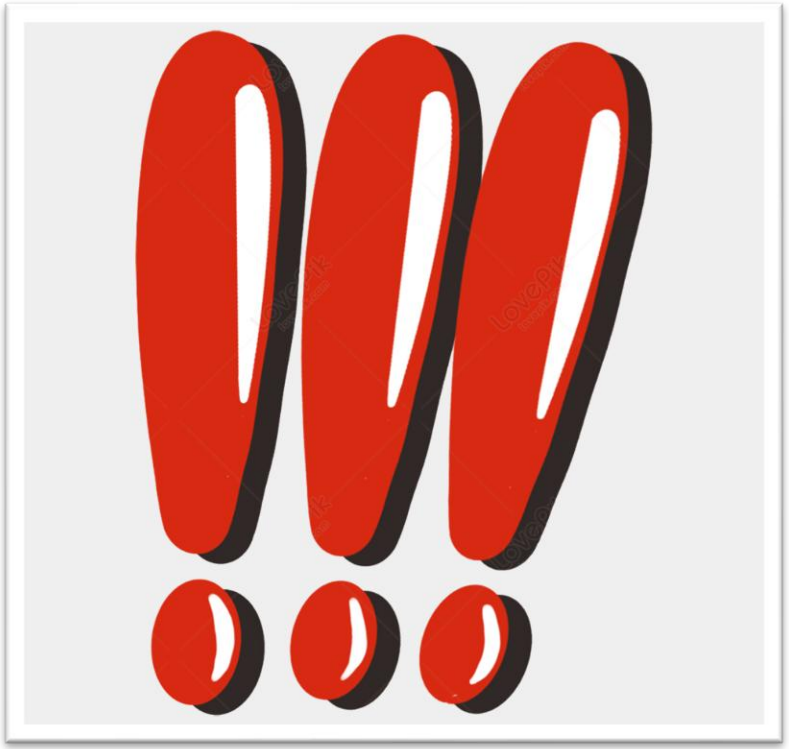
# Teaching & Learning in collaborative environment

**Knowledge is socially constructed** (Vygotskij).

Mathematical understanding grows through dialogue, collaboration, and the collective refinement of ideas, not in isolation, but within a community of thinkers.

# Some “products” from this field



For the pupils:

- Laboratory of Mathematics

For the teachers:

- Lesson Study
- Scuole Secondarie Potenziate in Matematica

# The Mathematics Laboratory

based on Giacardi (2011)

# Innovative – not recent

- John Dewey ( 1859-1952)
- Georg Kerschensteiner (1854-1932)
- Ovide Decroly (1871-1932)
- Edouard Claparède (1873-1940)
- Adolphe Ferrière (1879-1960)
- Maria Montessori (1870-1952)

# In Mathematics as well

- The most essential idea in the method of study called *Practical Mathematics* is that the student should become familiar with things before he is asked to reason about them (John Perry, 1913)

# Other experiences

- Eliakim Hastings Moore (US)

*mathematics is indeed itself a fundamental reality of the domain of thought, and not merely a matter of symbols and arbitrary rules and conventions*

- Jean Gaston Darboux & Émile Borel (France)

*amener non seulement les élèves, mais aussi les professeurs, mais surtout l'esprit public à une notion plus exacte de ce que sont les Mathématiques et du rôle qu'elles jouent réellement dans la vie moderne*

# Felix Klein (1849 – 1925)

- Line geometry
- Mechanics
- Algebraic geometry
- Mathematical physics
- Complex analysis
- Erlanger Programm (1872): the study of the properties of a space that is invariant under a given group of transformations

# Meraner Vorschlägen (1905)

- Redefinition of boundaries of secondary and higher teaching
- Functional thinking
- Application of mathematics
- Modelling
- Spatial intuition
- History and epistemology
- Connection with real problems and physics

# IV International Congress of Mathematicians

- Rome, 1908 - https://www.icmihistory.unito.it/1908-1910/ICM-1908.pdf
- Rich section on didactics: Gutzmer, Godfey, Eugene Smith, Suppantschitsch, Vailati, Fehr, Archenhold, Andrade, Conti, De Galdeano, Beke, De Amicis, Delitala
- The International Commission on Mathematics Instruction (ICMI) was founded, with Klein as first president

# Giovanni Vailati (1863 - 1909)

- Mathematician
- Historian
- Philosopher

Worked on the reform on *pre-fascism* Italian school:

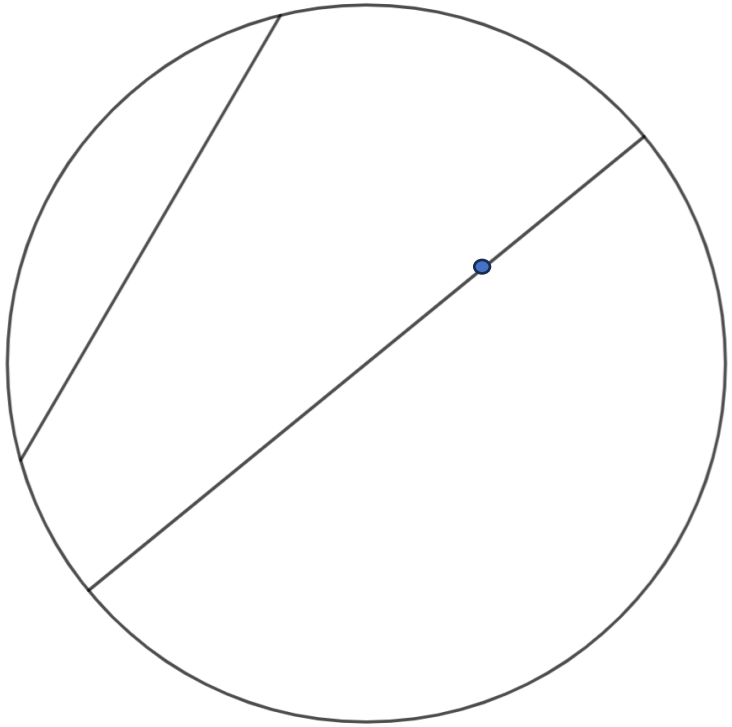
*[School is a laboratory,] a place where pupils are given the means to train themselves, under the guidance and advice of their teachers, to experiment and solve problems, […] to test themselves in the face of obstacles and difficulties designed to challenge their ingenuity and cultivate their initiative.*

# Giovanni Vailati (1863 - 1909)

*Guiding and encouraging pupils to acquire, through experimentation and, in particular, through the use of drawing instruments, as much factual knowledge as possible about how figures are constructed and their properties, especially those that are not "intuitive", is, on the other hand, the best way to awaken in them the desire and need to understand "how" and "why" these properties exist, and to predispose them to regard as interesting the learning or research of deductive connections between them, and of reasoning that leads to recognising them as consequences of one another.*

# Mathematics as a unit

Consider, for example, how much more easily a pupil would recognise the meaning and significance of a proposition such as this: that “the geometric mean of two numbers can never exceed their arithmetic mean”, if it were pointed out to them that, in a circle with a diameter equal to the sum of two segments, the second is represented by the radius, and the other by half of a chord.



# The «modern» laboratory of mathematics

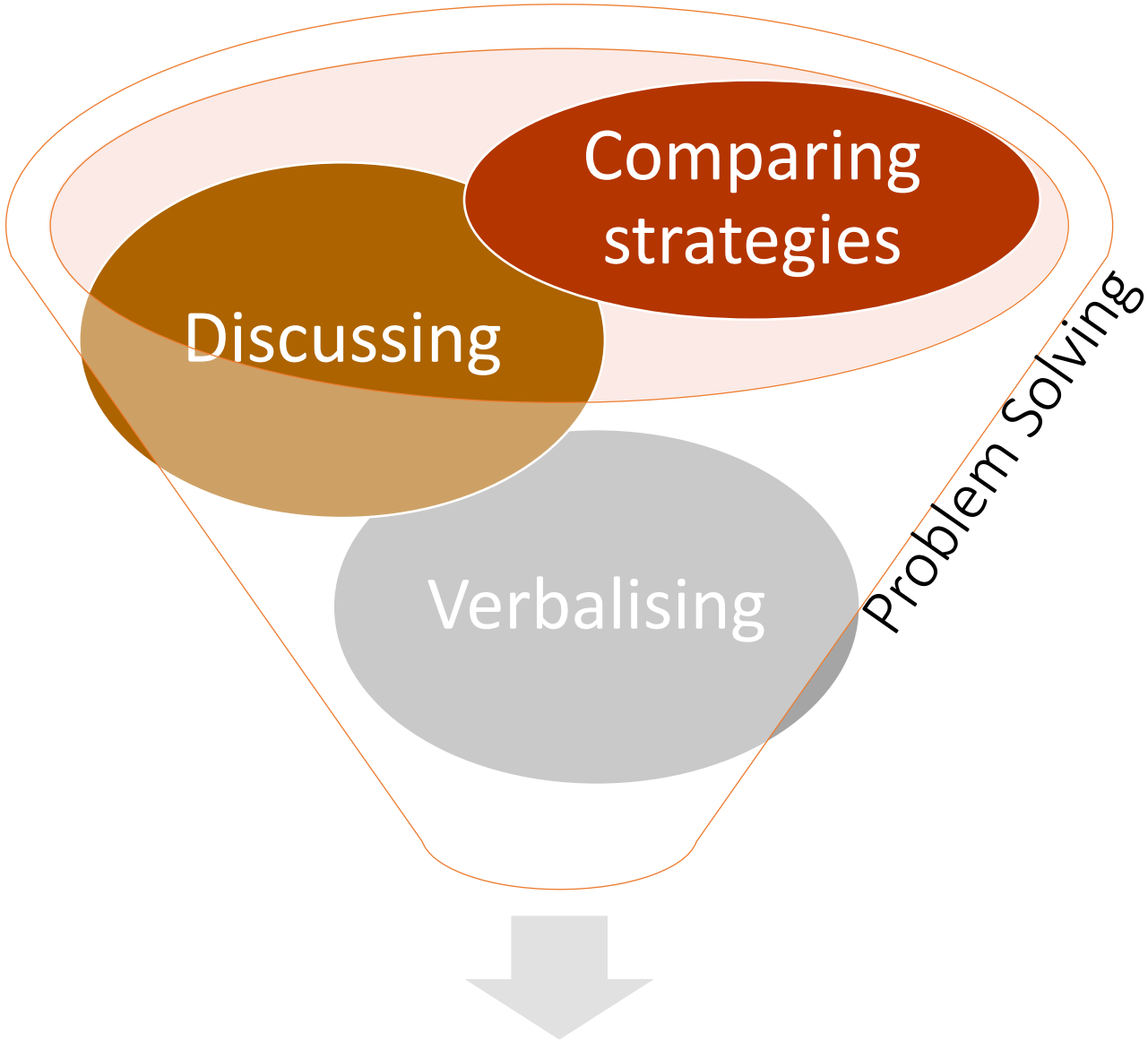
The mathematics laboratory is not a physical place separate from the classroom, but rather a structured set of activities aimed at constructing the meanings of mathematical objects. The laboratory therefore involves people (students and teachers), structures (classrooms, tools, organisation of space and time), and ideas (projects, teaching activity plans, experiments).

# Mathematics as a tool for citizenship

- Which allows the subject to return to what has been done, seen (etc.), producing interpretations, explanations, answers to questions such as “why is it so?”

- Which allows the subject to anticipate events, situations, etc., producing predictions, hypothetical discussions about possible worlds, answers to questions such as “what will it be like?”, “what could it be like?”

(from *Matematica per il Cittadino 2021*)




# Laboratory of Mathematics

# Hands-on: task one

Cutting angles


From a seminar by Lambrecht Spijkerboer

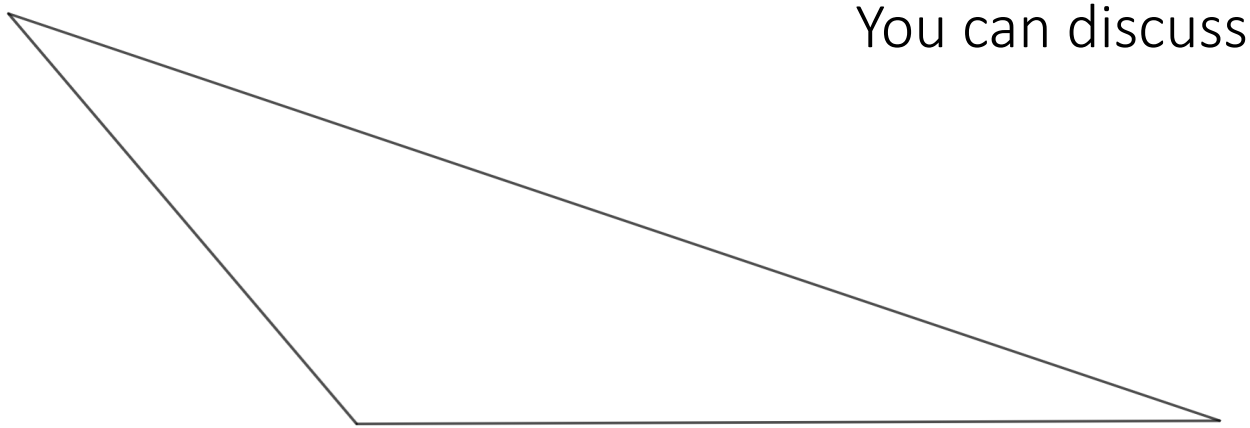
# Task 1

Cut this obtuse-angled triangle in acute-angled triangles.

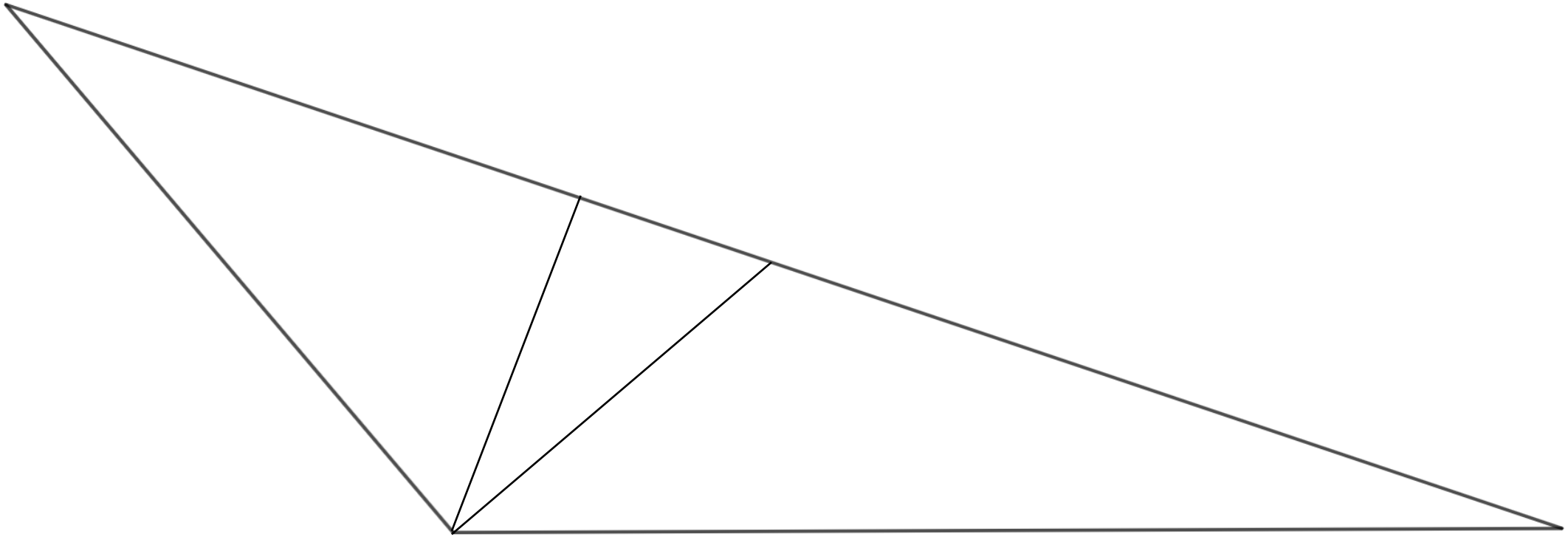
5’ individual work

15’ group work

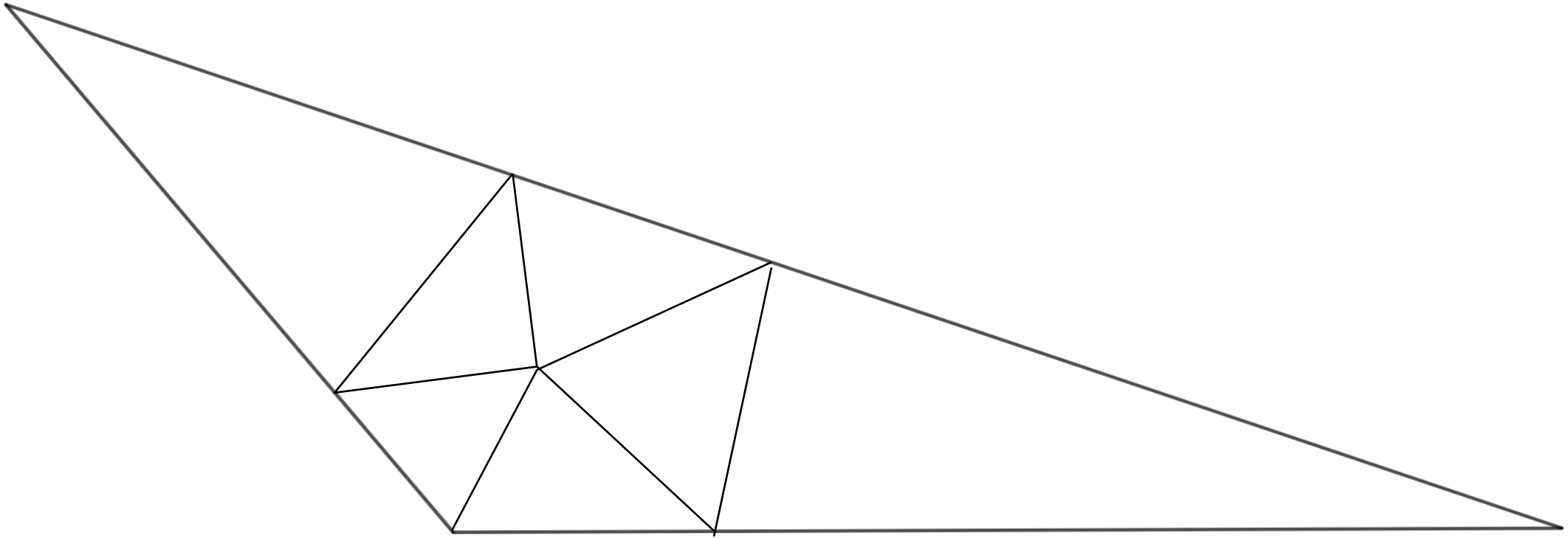
You can discuss in Polish



# Impossible?



# possible !!!



# Hands-on: task two

Origami triangle

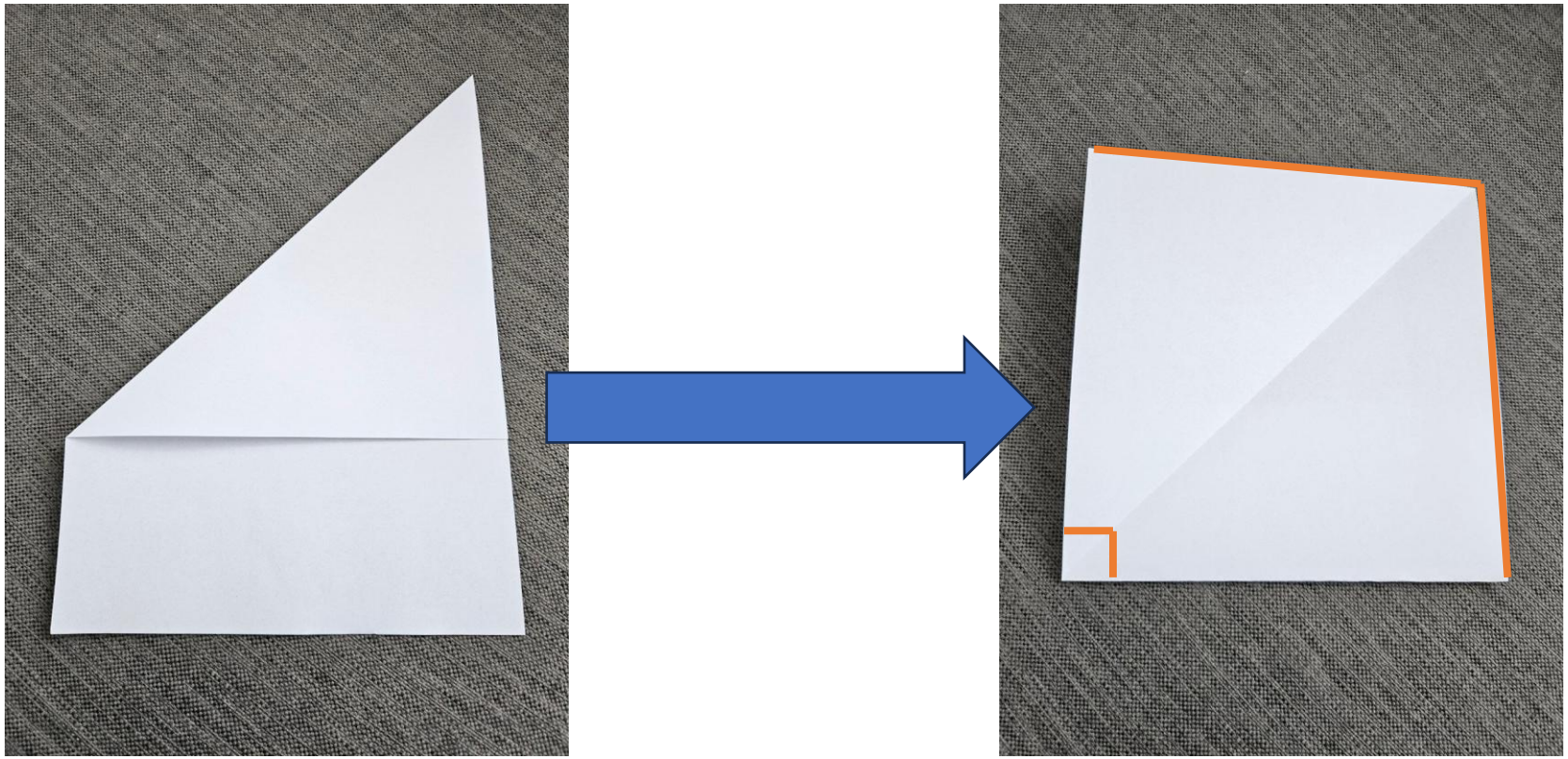

From a seminar by Maria Gemma Gallino & Stefania Serre

# Step 1

Take the blank sheet.

Cut from it the biggest square possible.
How do we do it?

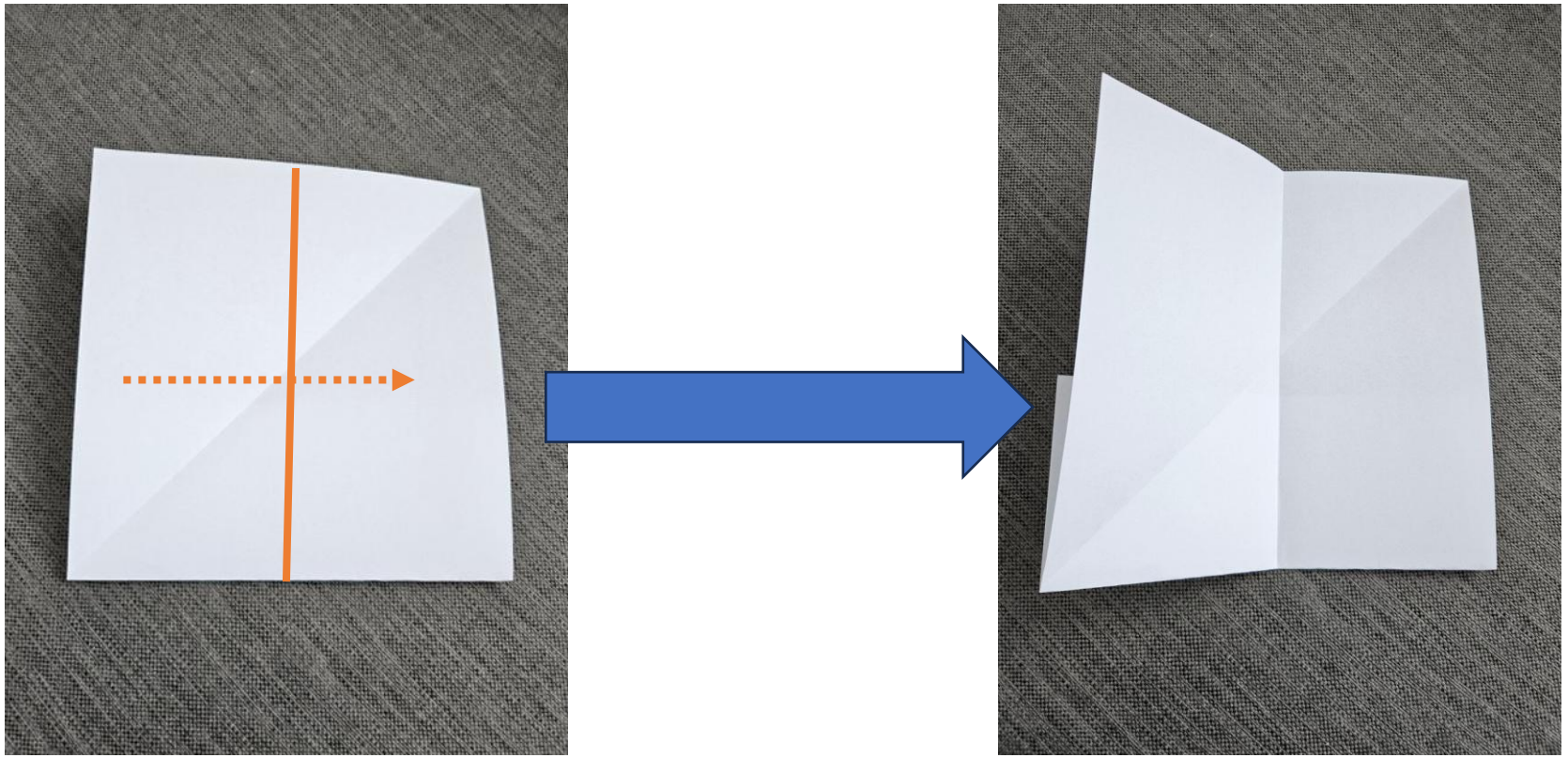
# Step 1



This is a square because these two sides overlap (therefore have the same length) and the angles are right angles by construction.

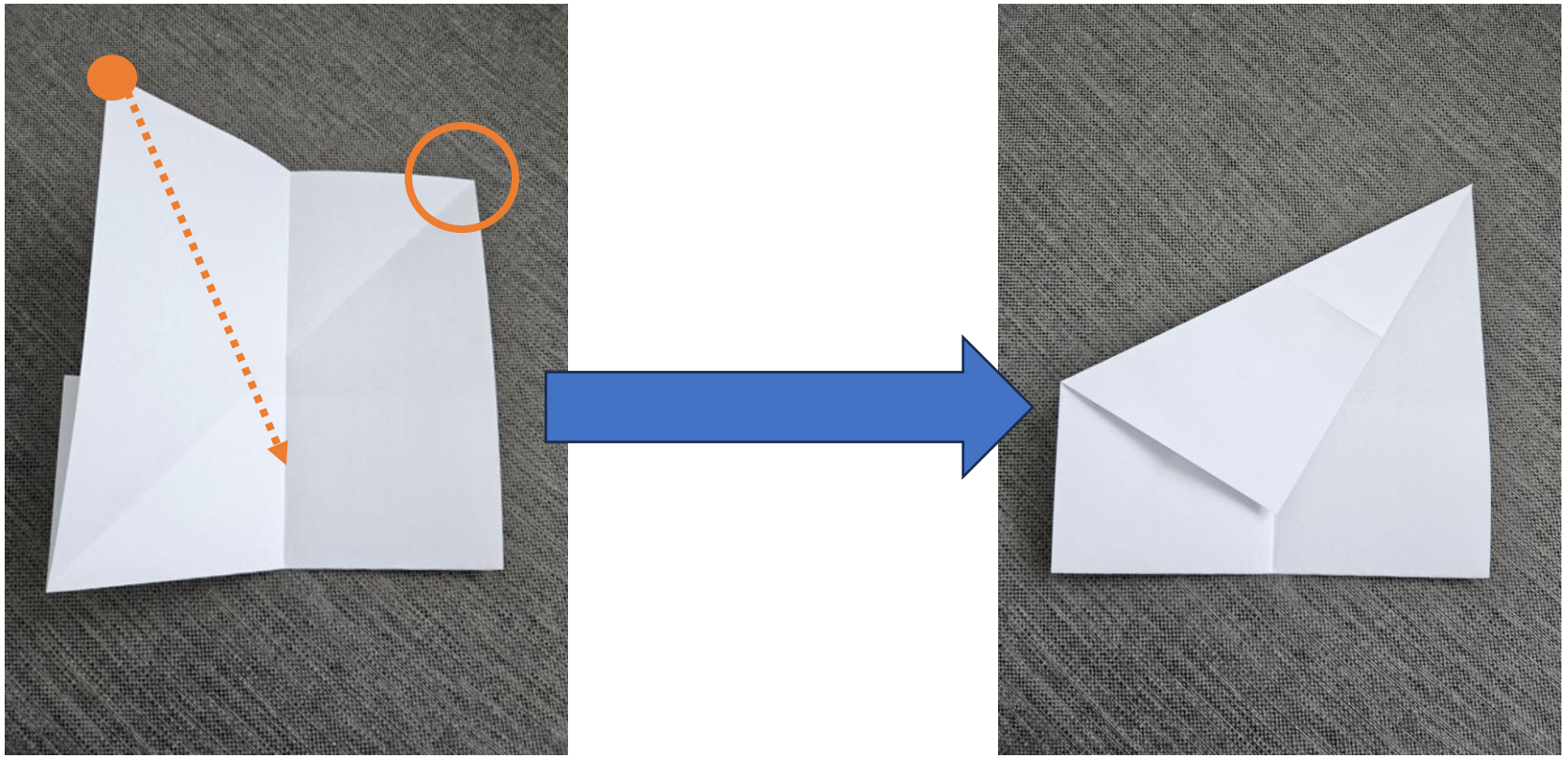
# Step 2

Fold the square in half, on the mid-point of two opposite sides.



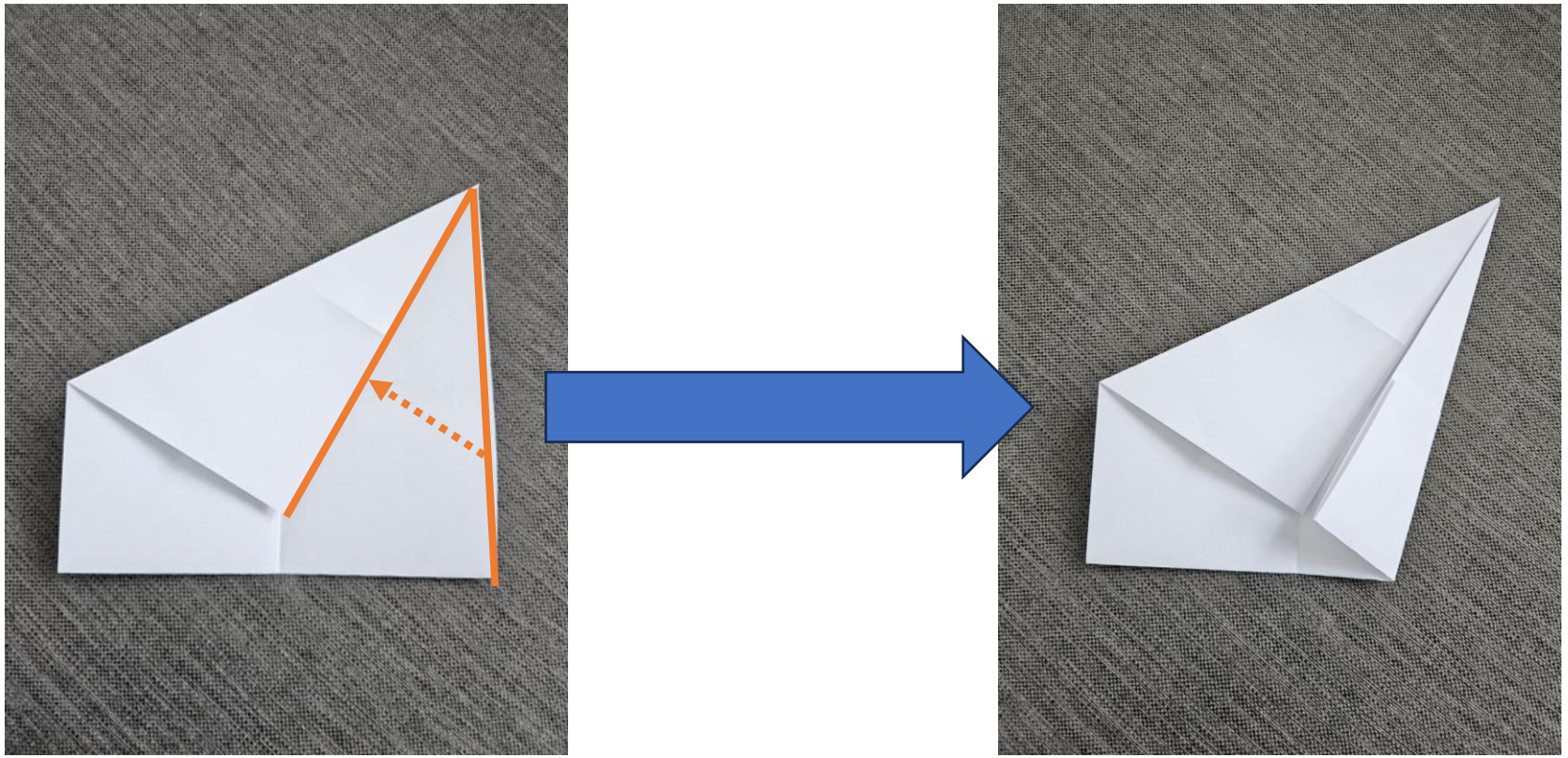
# Step 3

Cut this angle by folding this vertex on the previous folding line.



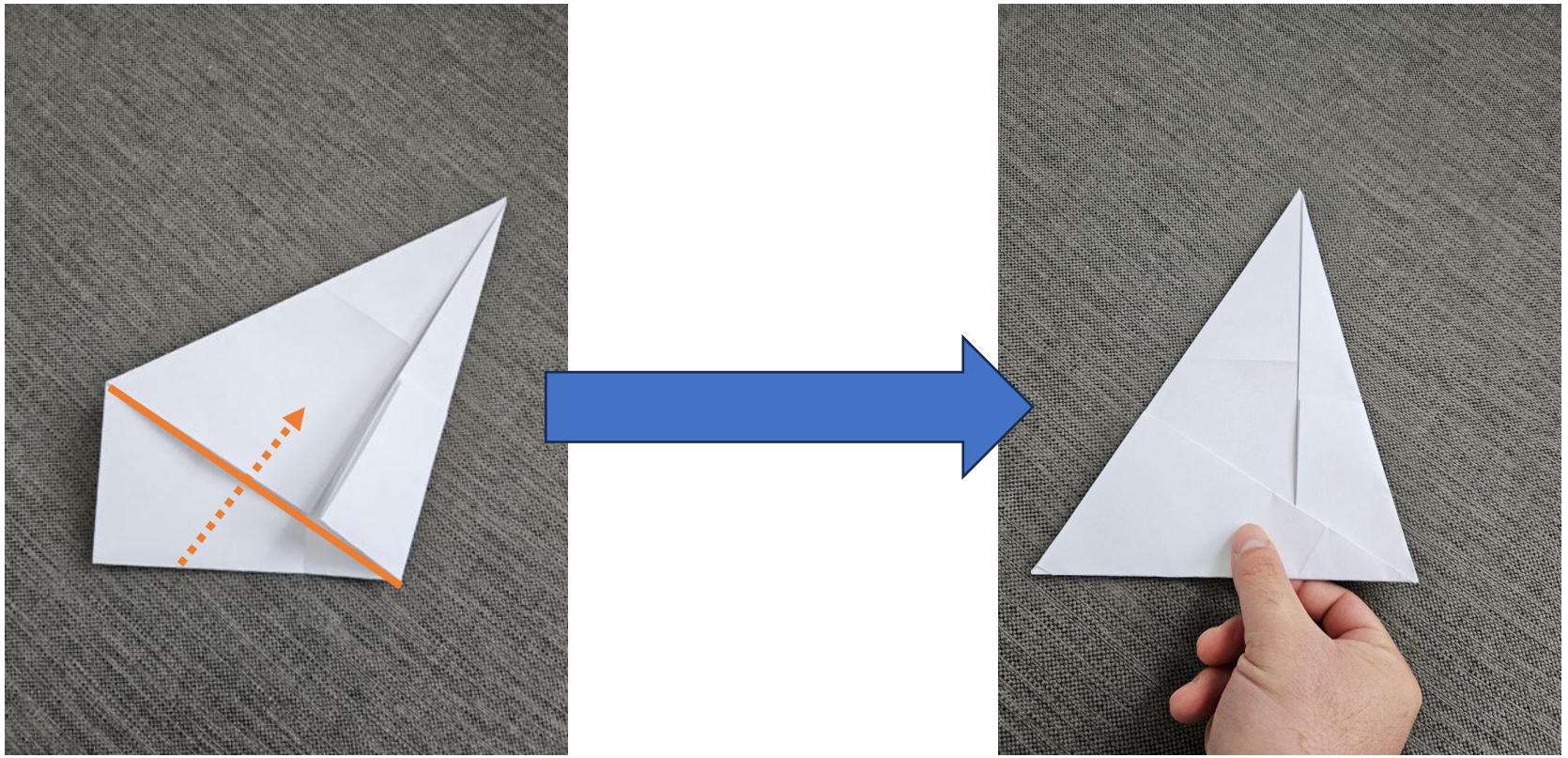
# Step 4

Overlap these two sides.

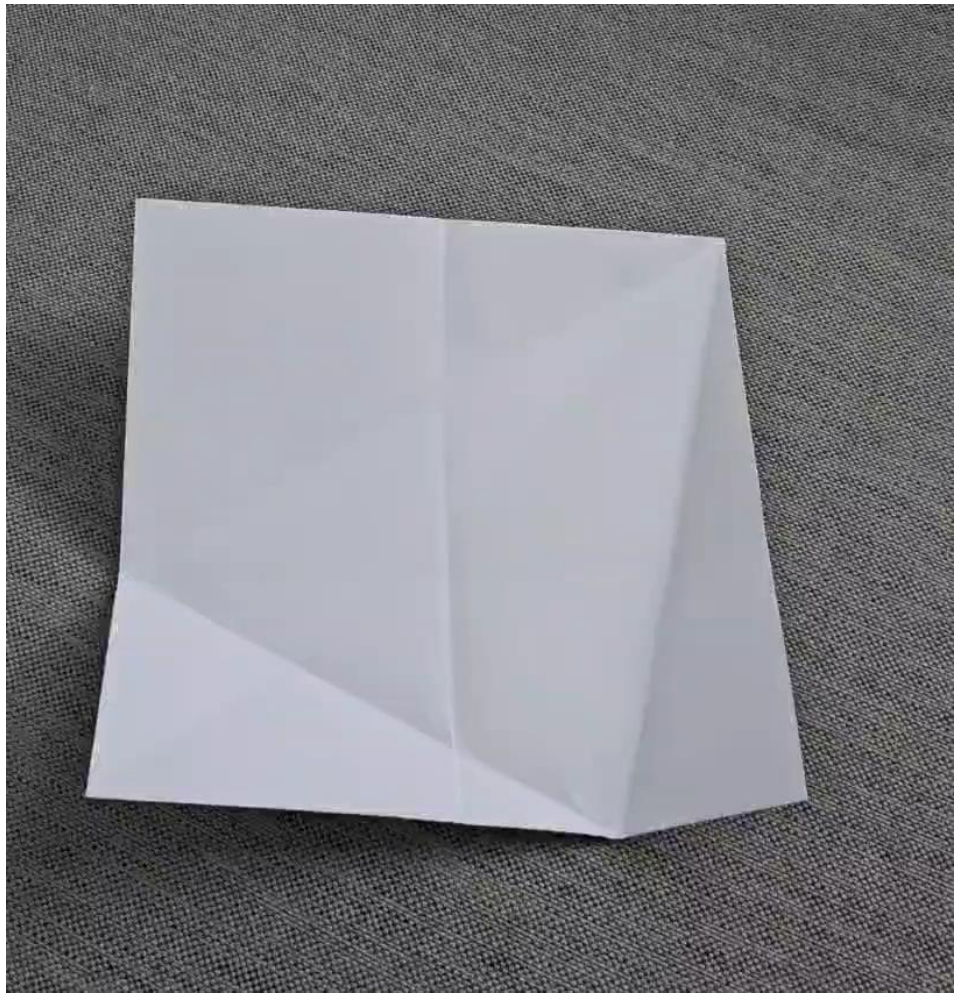


# Step 5

Finally, fold over this side.



# Pit-stop for a check, in case you need some help



# Task 2

Find the measure of each angle of this triangle. Justify your answers by adequately explaining all the steps you went through. You can use the geometrical properties of the folding that we did.
No protractors and no eyeballing.
You need to be able to explain your reasoning to the other groups.

10’ group work

Elect a representative to present your answer

You can discuss in Polish

10’ for group presentations

# Group presentations

We listen to the presentation of each group.

We take notes of steps that are correct, but different from the ones we followed.

We take notes of steps that do not convince us, and think why they do not convince us.

We discuss our observations.

# What is the point?

Developing social skills:

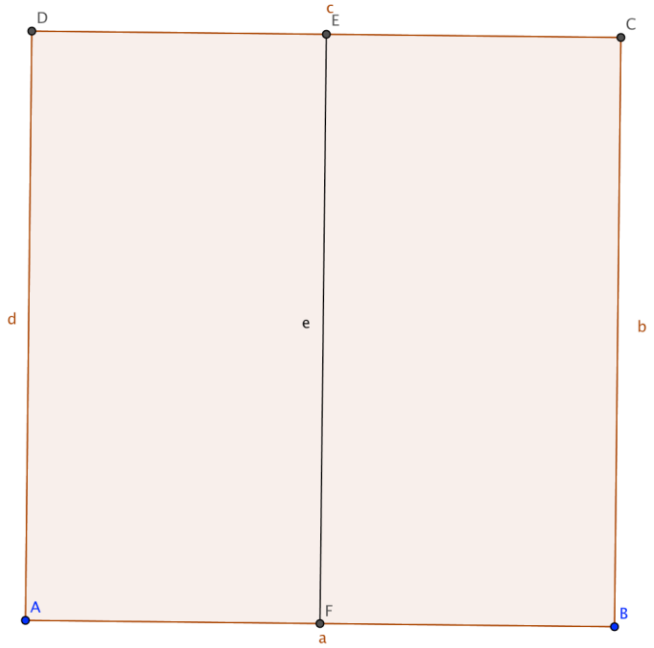
- listening
- discussion
- argumentation

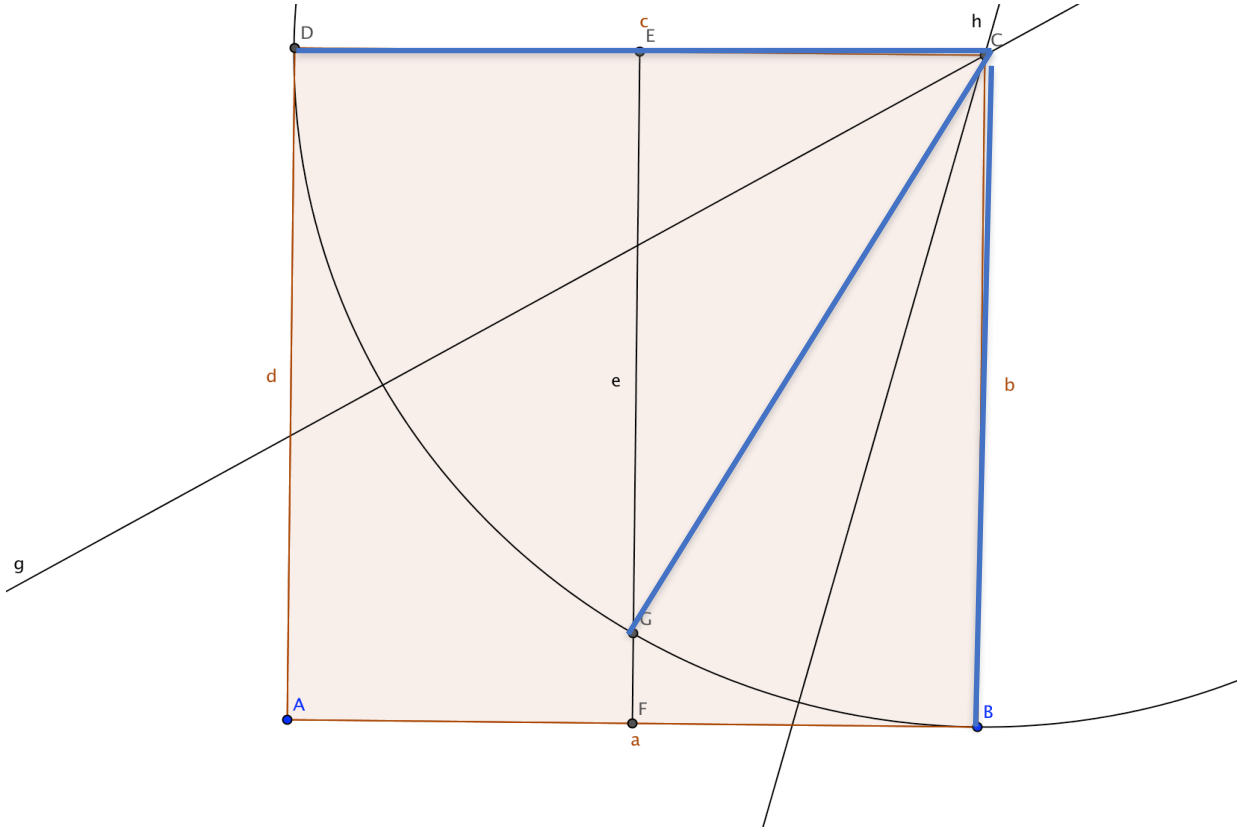
Developing mathematical skills:

- mathematical modelling
- abstraction
- proof

Evolving knowledge: previous knowledge is reapplied in an unknown situation to develop problem solving

# Further development: GeoGebra



# Before we finish, a question for you

https://forms.gle/aoSdXPobEsTPzaUb9

# References

Giacardi, L. (2011). L'emergere dell'idea di laboratorio di matematica agli inizi del Novecento. *Il Curriculum Di Matematica e Fisica Nella Scuola Del III Millennio: Infanzia, Primaria, Secondaria Di Primo e Di Secondo Livello*, 55–65.

Vygotskij, L. S., & Robbins, D. (1999). *The collected works of L.S. Vygotsky: Volume 6, Scientific legacy* (R. W. Rieber, Ed.; M. J. Hall, Trans.; Softcover reprint of the hardcover 1st edition 1999). Springer Science Business Media, Llc.

Thank you very much!

ありがとうございました

Grazie mille!

Dziękuję bardzo!