In the laboratory of Mathematics

Algebraic Inquiry

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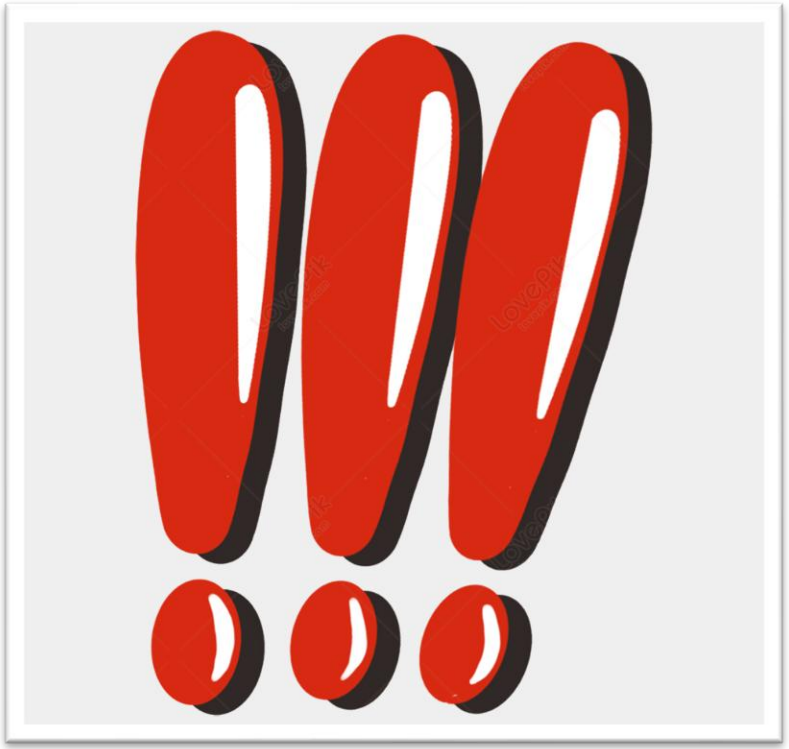
Some “products” from this field



For the pupils:

- Laboratory of Mathematics

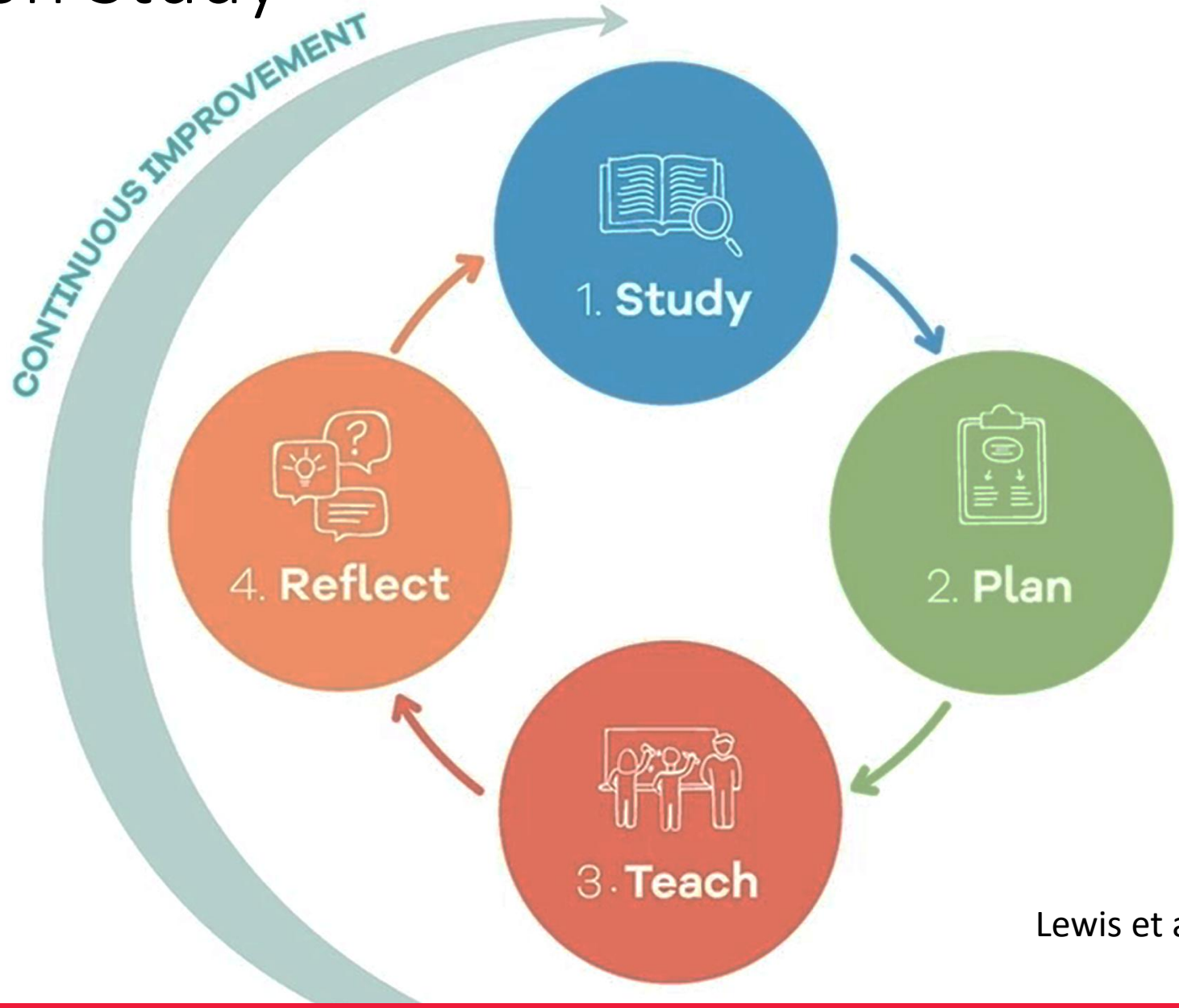
For the teachers:

- Scuole Secondarie Potenziate in Matematica
- Lesson Study

Lesson Study

Teachers collaborating for task design

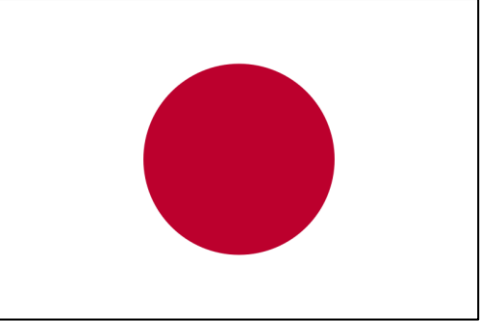
What is Lesson Study



Lewis et al. (2019)

Phases of Lesson Study

1. **Study**: Students' learning goals are decided, and a *research topic* is set to be investigated in relation to such goals. Relevant teaching materials are analysed;
2. **Plan**: A detailed Lesson Plan for a *research lesson* is designed;
3. **Teach**: The research lesson is taught in a real classroom, and observed by other teachers;
4. **Reflect**: The research lesson is discussed, based on data collected during the observation, especially on students' learning.



Lesson Study

One example of task *from* collaboration

Miyakawa and Winsløw (2019)

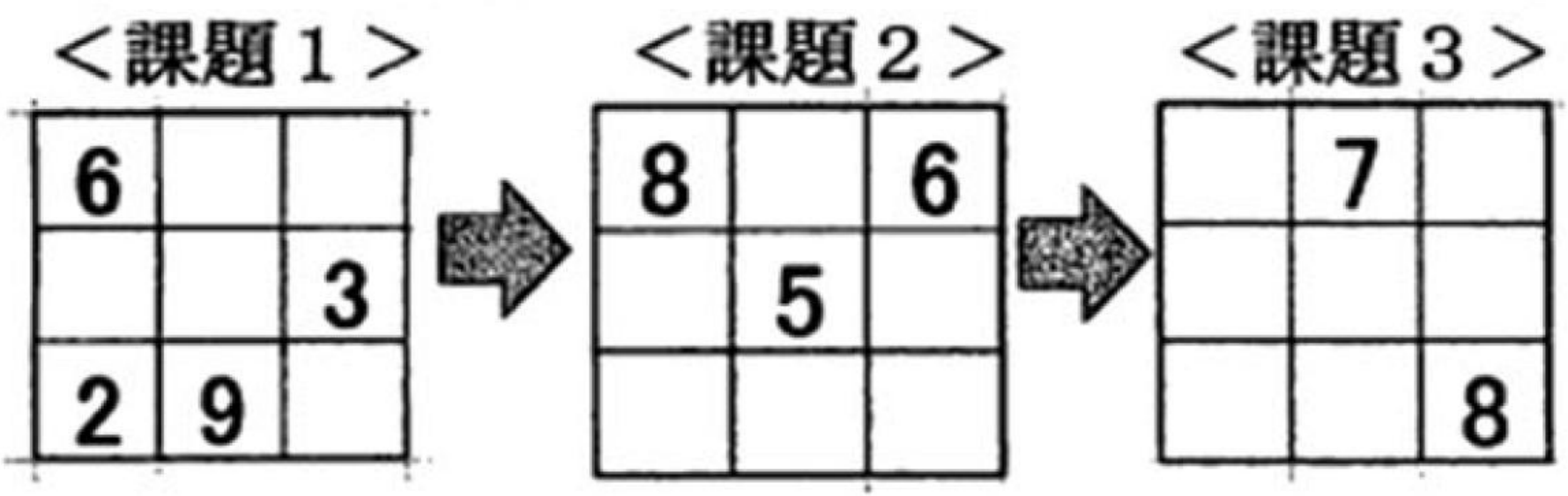
Introducing linear equations

- Research problem: *introducing linear equations* through problem solving
- Teaching idea: progressively introduce the need for mathematical modelling
- Methodology: open problem solving
- Grade 7

Magic square

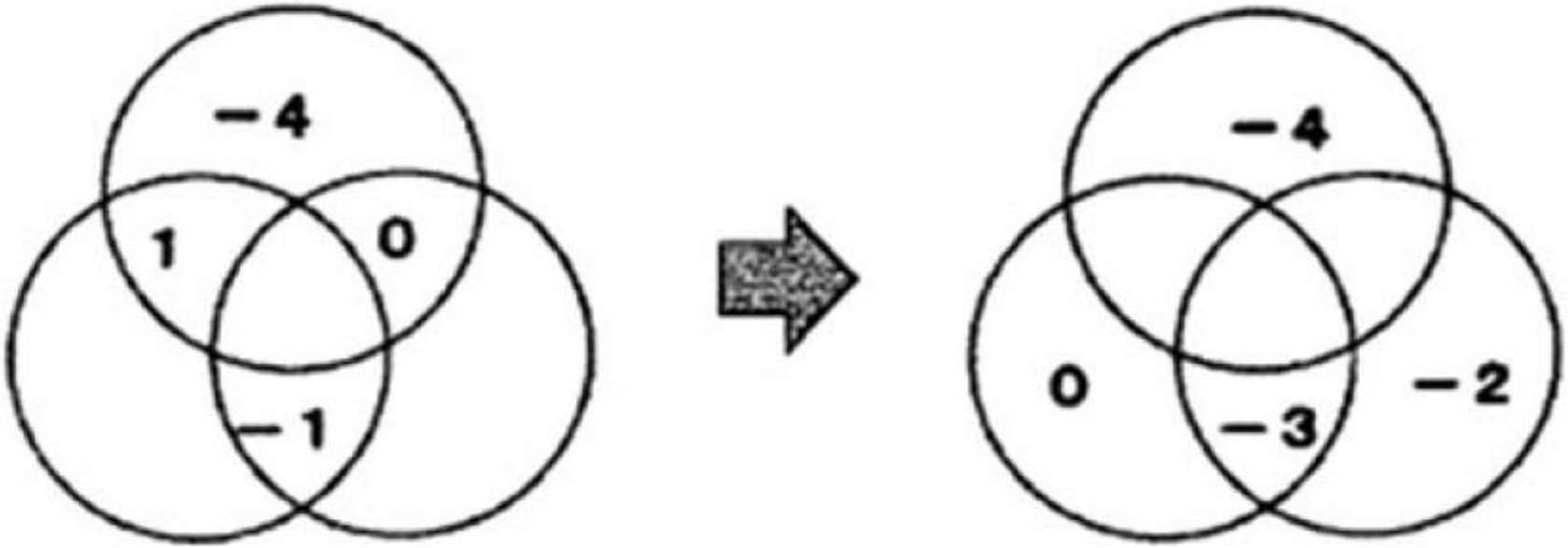
- Progressive assignment:

the magic number for each magic square is 15



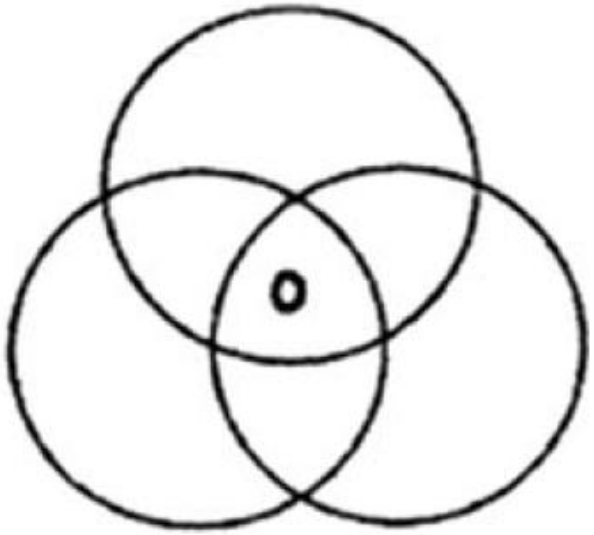
Magic circle\1

- Fill the empty space so that the sum of the numbers in each circle is the same



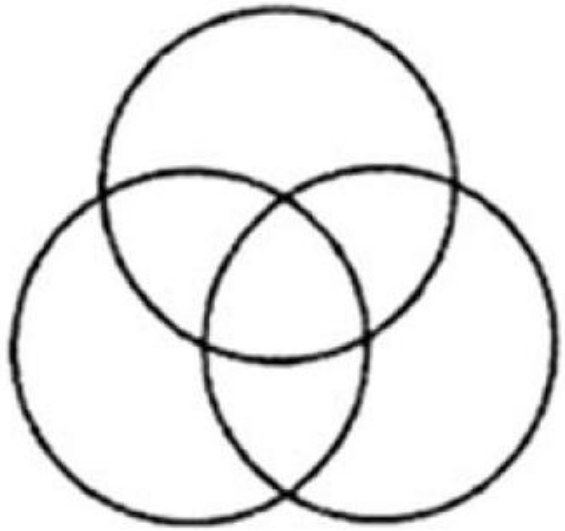
Magic circle\2

- Complete the following magic circle



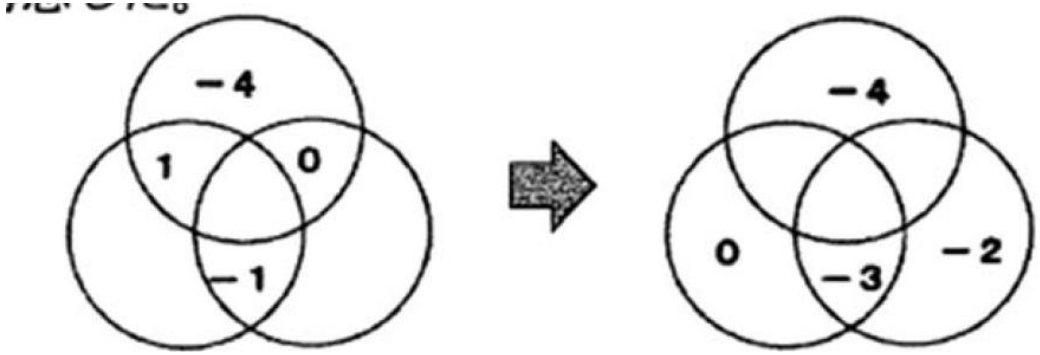
Magic circle\3

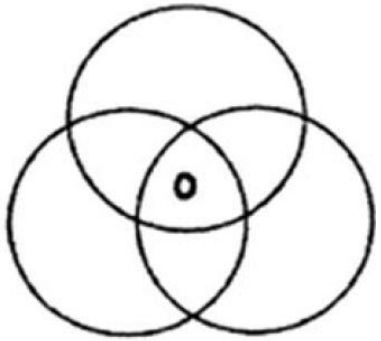
- Create your own set of magic circles and share it with the class as a problem to solve

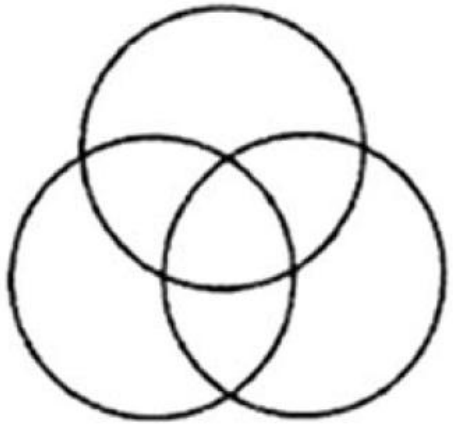


Pause for thought

What Mathematics are we working with?







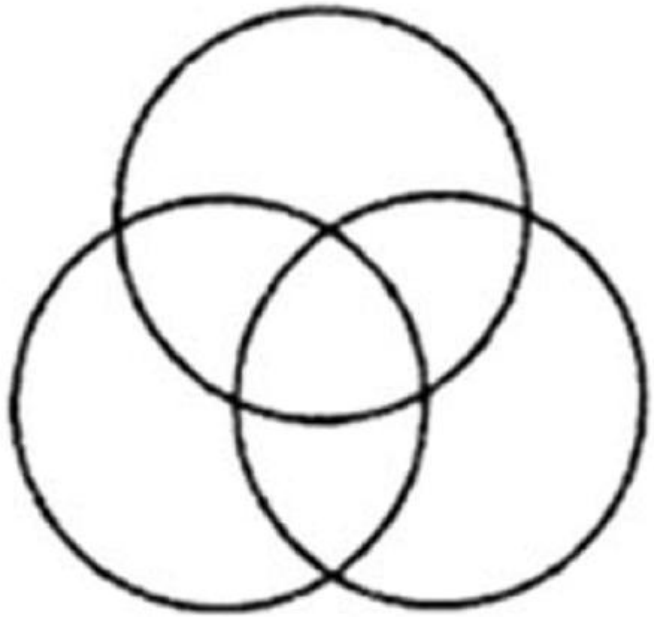
Introducing linear equations (???)

- Unclear connection between the tasks and the topic
- At the introductory stage, students will not see the merit of using equations

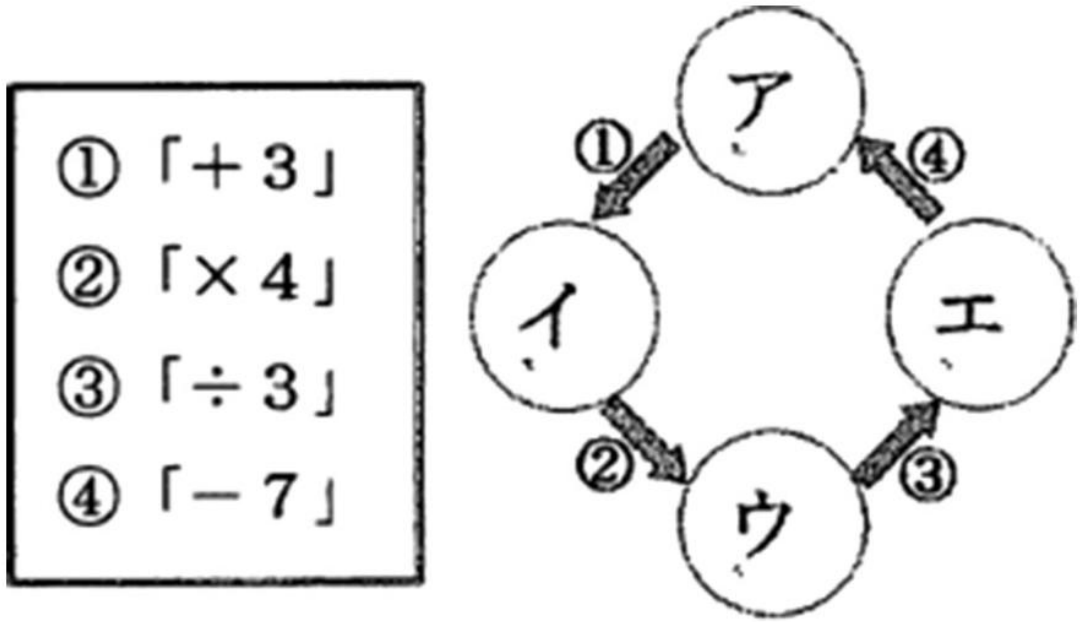
Not really an “introduction”

- The role of the central number is not intuitive
- The model involves simultaneous equations

$$\begin{cases} a + f = b + e \\ c + f = b + d \end{cases}$$

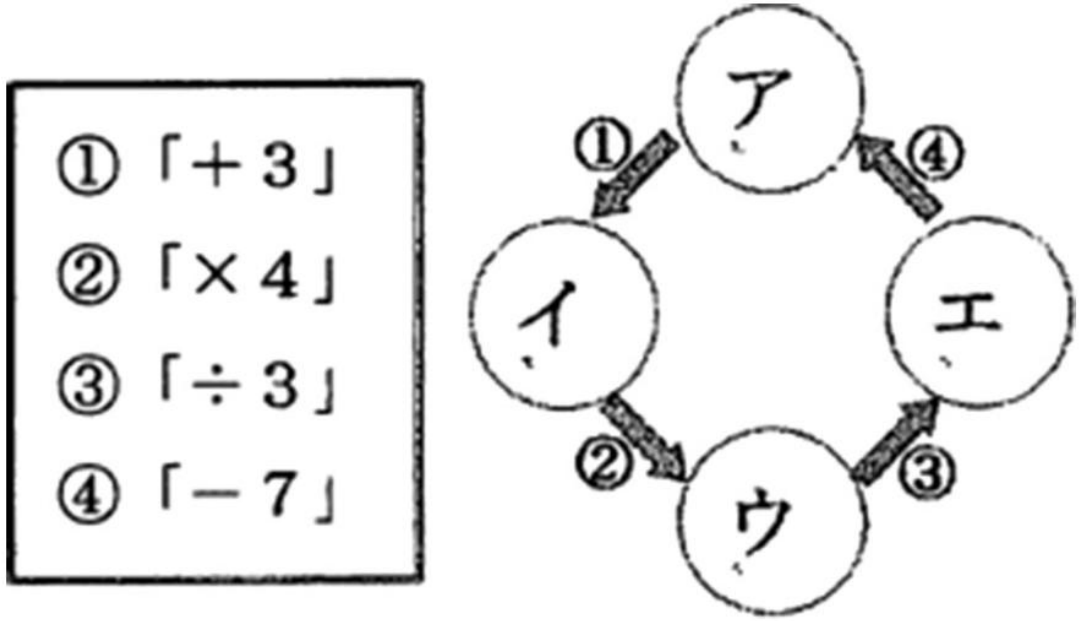


Another idea: calculation square

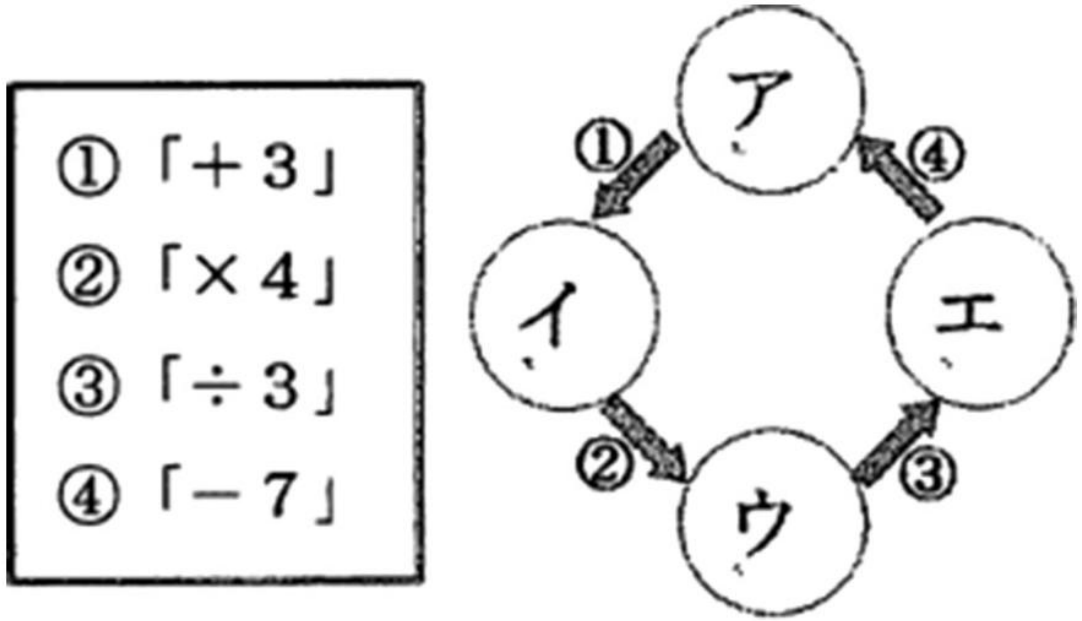


Pause for thought

Why would this work better?



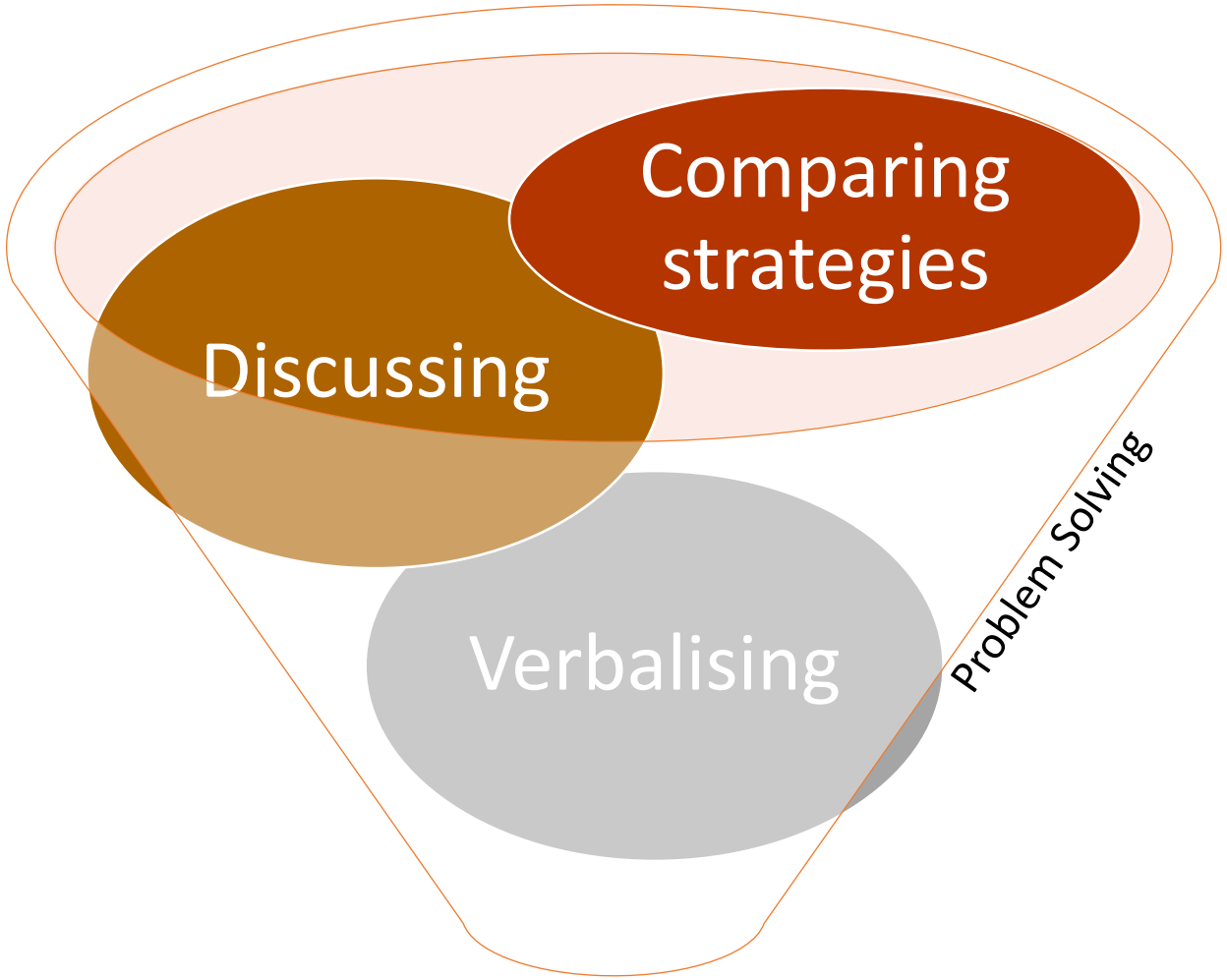
Another idea: calculation square



From trial & error to $\frac{4(x+3)}{3} - 7 = x$

Scuole Secondarie Potenziate in Matematica

Let’s go back inside the laboratory of mathematics



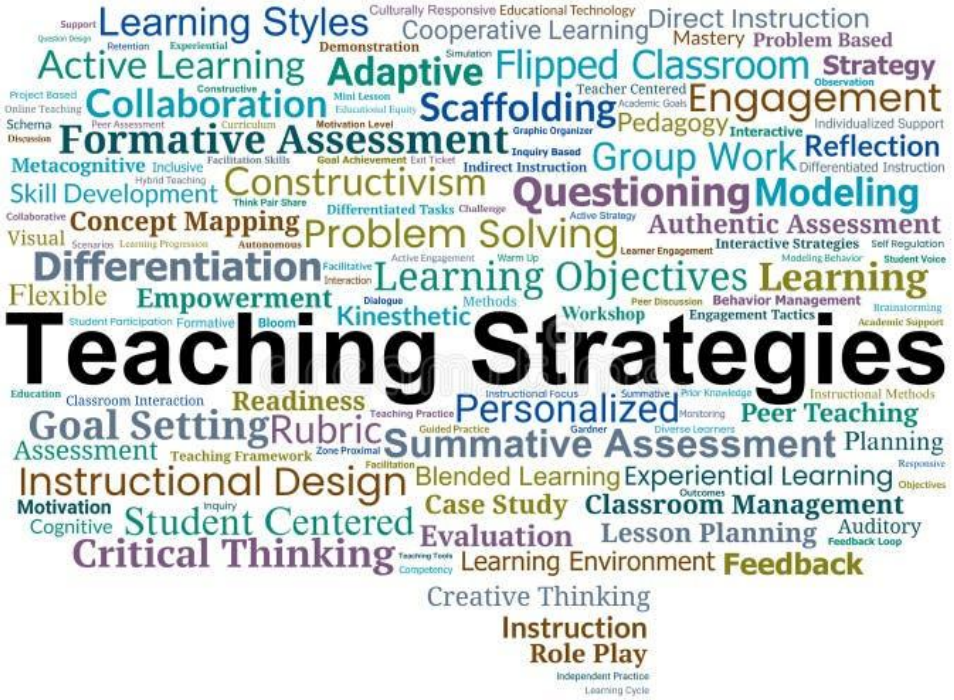
Laboratory of Mathematics

=

social construction of knowledge (Vygotskij)

The laboratory is a container

Inside the laboratory of mathematics, we can experiment different methodological approaches to teaching and learning.



Method of Varying Inquiry

(or Variation Method)

Inquiry

1. Problematic situation: observe (O_i), ask questions (Q_j), provide answers (A_k)
2. Change one (or more) O_i usually by negation → the situation changes ($\sim O_i$)
3. New observations are born (O_i*), together with new questions (Q_j*) and new answers (A_k*)

- Why is it so?
- What happens if this is not so?

Method of Varying Inquiry (MVI)

By *varying* the situation, *understanding* is improved.

We understand more when we observe from different viewpoints. We change the foundations one by one, and observe what happens.

(Swidan et al., 2023)

Founded on classic Chinese pedagogy

- CONTRAST: To experience something, one must experience something different to compare.
- GENERALISATION: To understand what “3” is, one needs to experience a variety of situations in which “3” appears.
- SEPARATION: To experience some aspect of something and to separate this from other aspects, one must vary it while the others remain unchanged.
- FUSION: If there are various critical aspects that learners must consider together, they must experience them simultaneously.

(Mellone et al., 2019)

MVI promotes hypothetical thinking

Argumentative activities in which hypotheses are produced or conditionalities are generated can be traced back to two main modalities […]

The first one is characterized by the production of interpretative conjectures about what is seen (perceived), e.g., to organize it.

The second one is characterized by the production of predictive conjectures (e.g., hypotheses about a future situation).

(from *Matematica per il Cittadino 2021*)

Hypothetical thinking as a tool for citizenship

- which allows the subject to return to what has been done, seen (etc.), producing interpretations, explanations, answers to questions such as “why is it so?”

- which allows the subject to anticipate events, situations, etc., producing predictions, hypothetical discussions about possible worlds, answers to questions such as “what will it be like?”, “what could it be like?”

(from *Matematica per il Cittadino 2021*)

Inquiry in Algebra

This activity proposes a series of stimuli that aim to explore relationships using algebraic language.

These are activities that encourage argumentation and demonstration using the symbolic language of natural numbers.

Your task

- Work with your neighbour to solve as many tasks as possible, and share your observations about:
  - The applications of this task in the classroom: when to do it, why, how it connects to the curriculum, what students need to know to approach the tasks…
- 20 minutes sharing with the whole group

References

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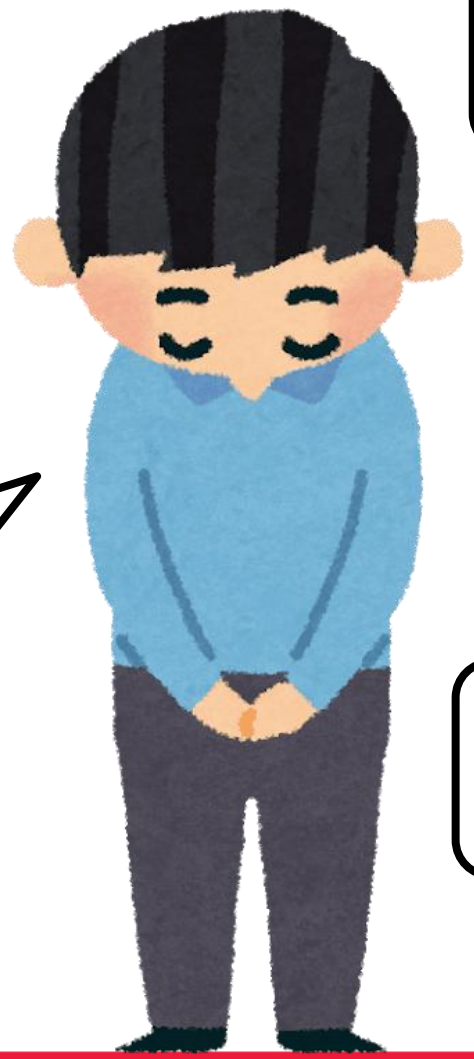
Mellone, M., Ramploud, A., Di Paola, B., & Martignone, F. (2019). Cultural transposition: Italian didactic experiences inspired by Chinese and Russian perspectives on whole number arithmetic. *ZDM*, *51*(1), 199–212. https://doi.org/10.1007/s11858-018-0992-7

Swidan, O., Cusi, A., Robutti, O., & Arzarello, F. (2023). The method of varying inquiry for stimulating learning. *For the Learning of Mathematics*, *43*(1), 14–18.

Website of Liceo Matematico - https://www.liceomatematico.it/
Website of Scuole Secondarie Potenziate in Matematica - https://difima.i-learn.unito.it/course/view.php?id=159

Thank you very much!

ありがとうございました



Grazie mille!

Dziękuję bardzo!