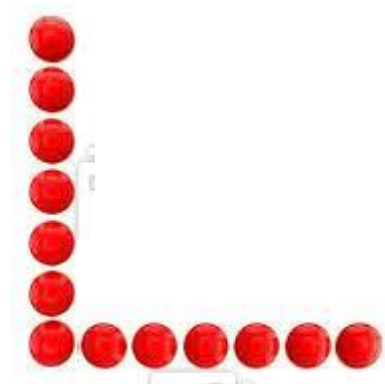


ARYTMETYKA Z KULEK

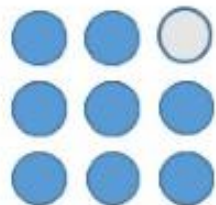
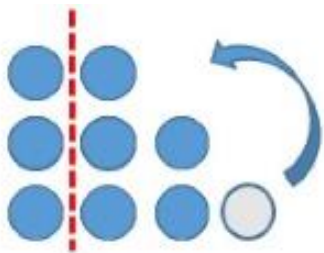
Liczby parzyste i nieparzyste



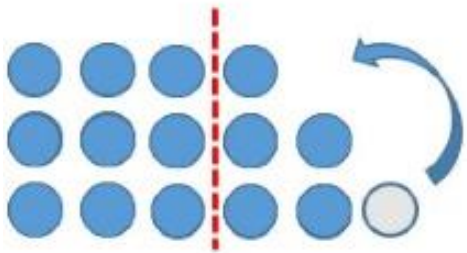
Parzystość sumy liczb

Suma trzech
kolejnych liczb

jest liczbą prostokątną
o boku 3



$$2 + 3 + 4 = 3 \cdot 3$$



$$4 + 5 + 6 = 3 \cdot 5$$

Liczby kwadratowe

1



4



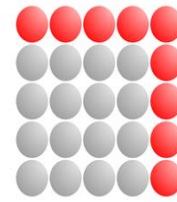
9



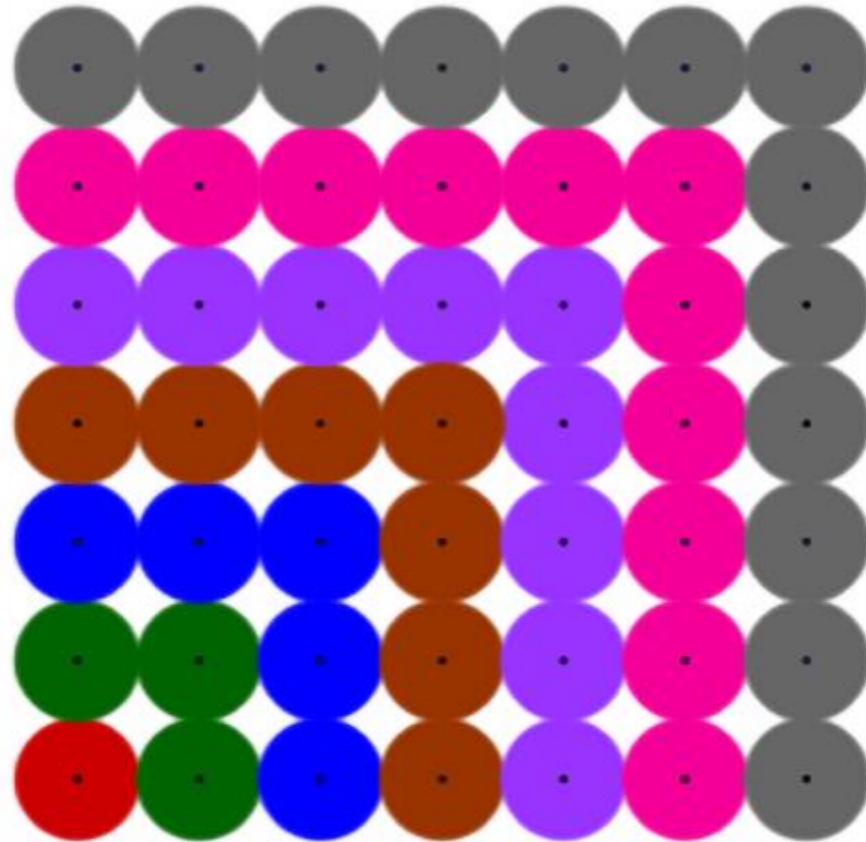
16



25



Suma kolejnych liczb nieparzystych



$$1+3+5+\dots+117 = ?$$

Liczby trójkątne

1



3



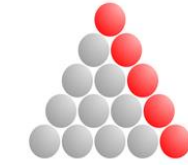
6

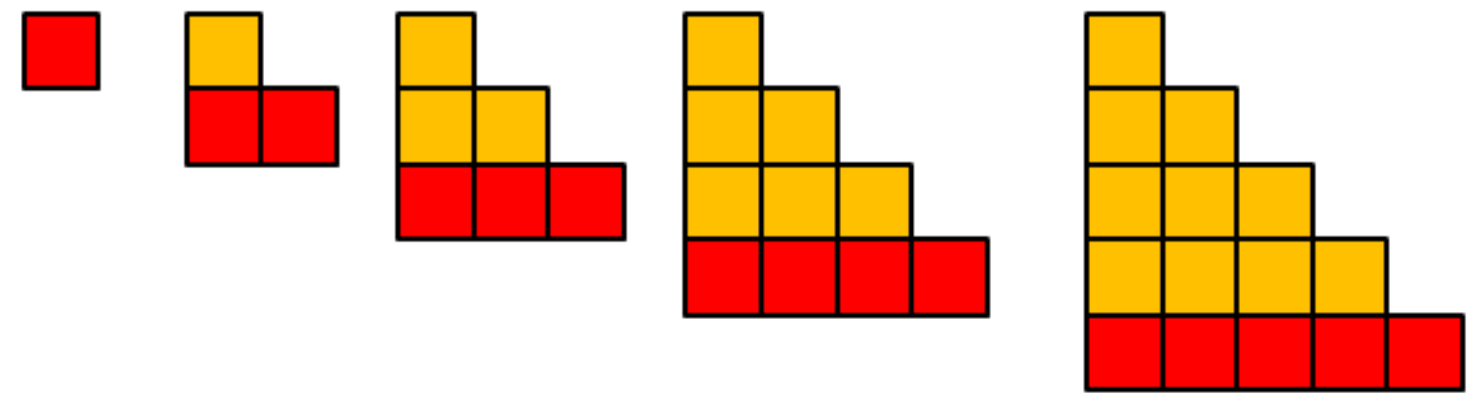
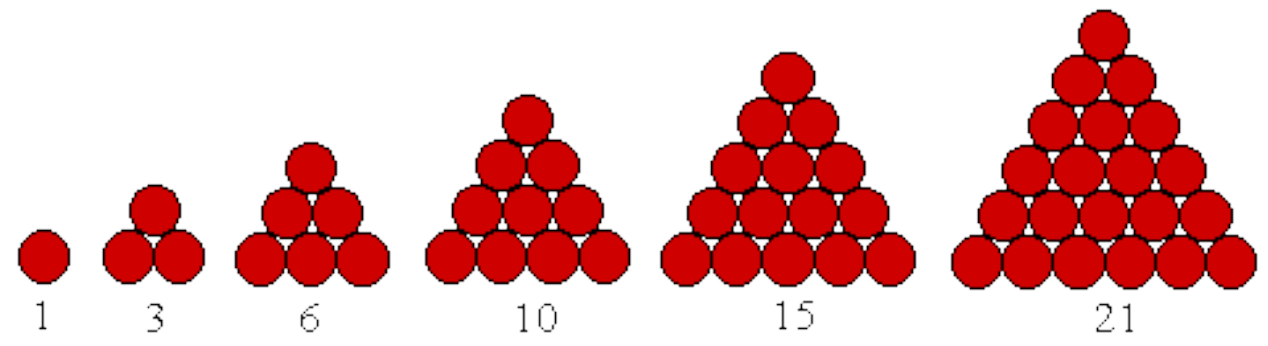


10



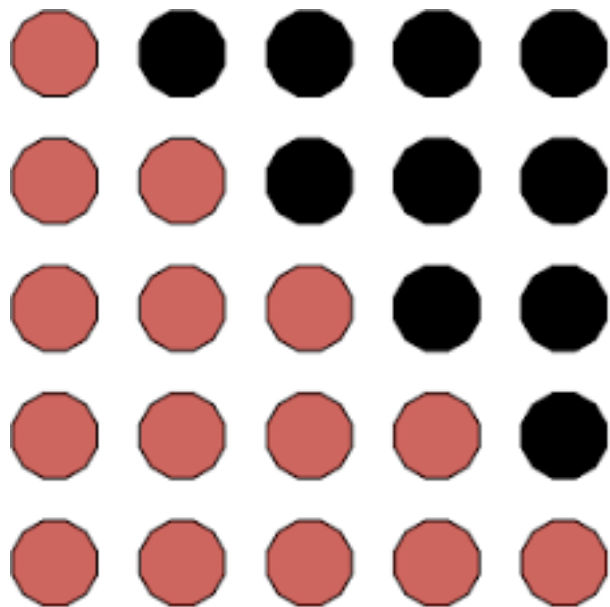
15

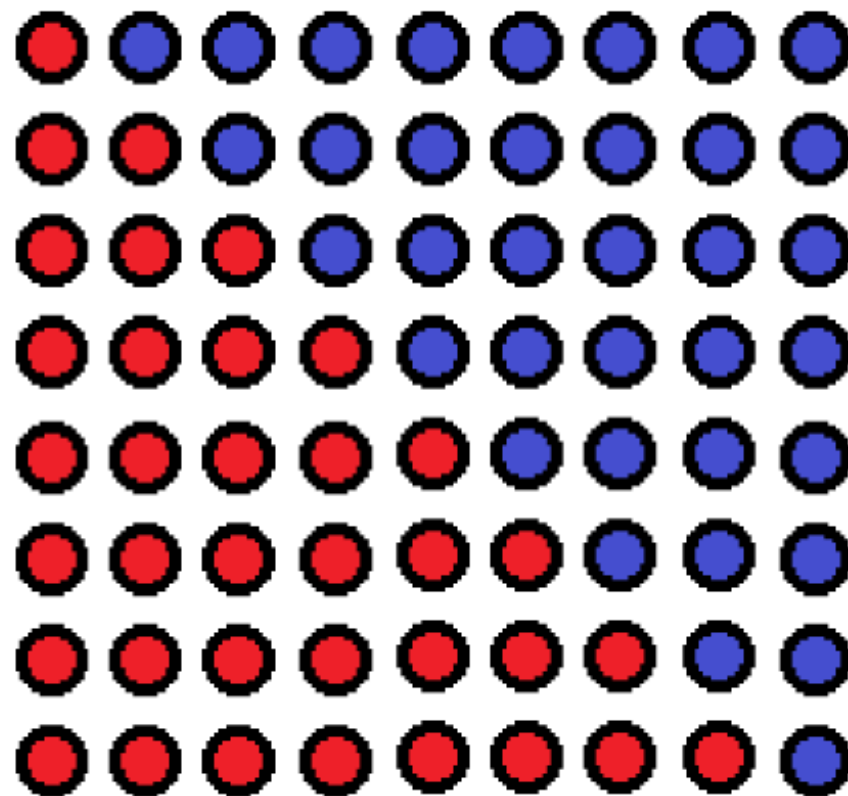




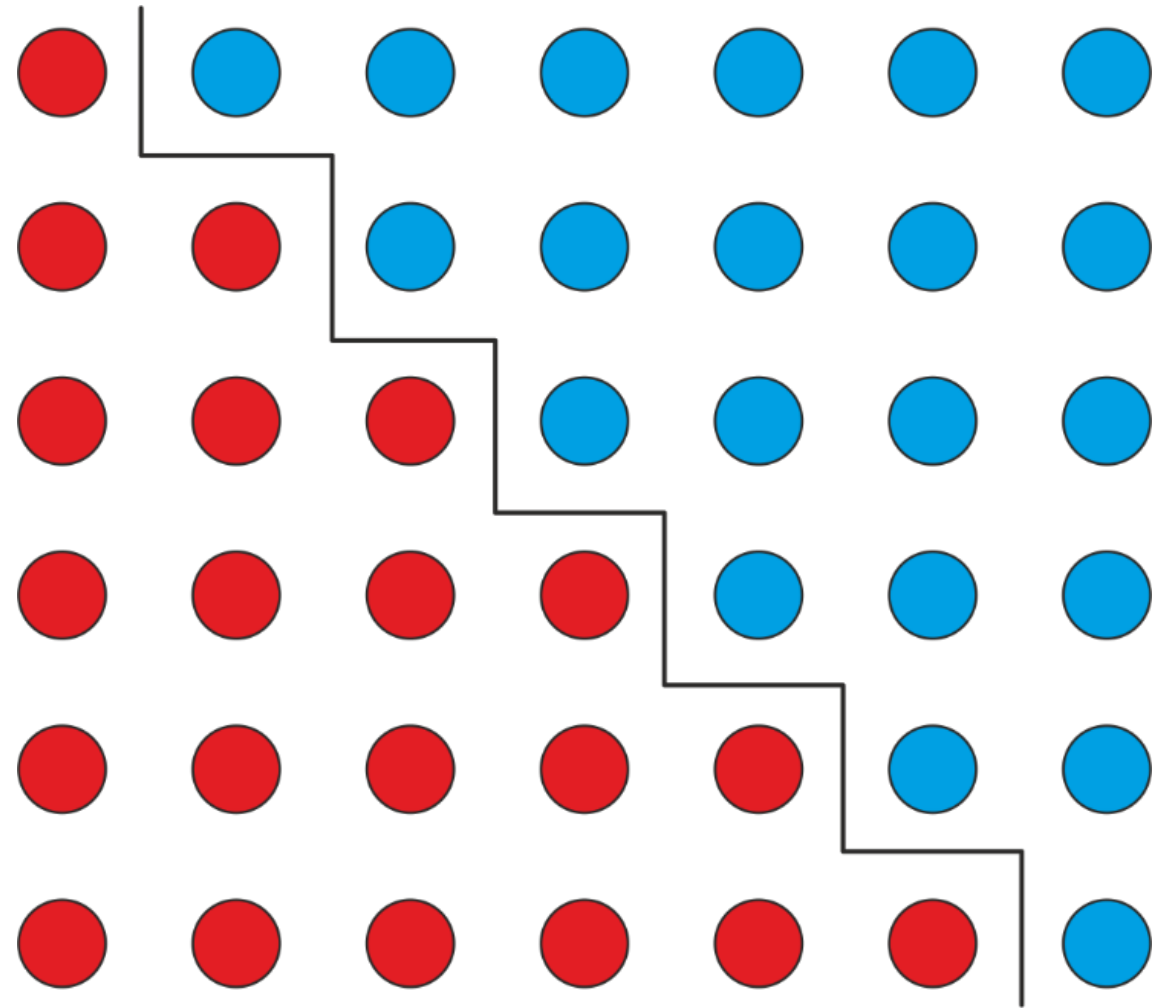


Suma dwóch kolejnych liczb trójkątnych

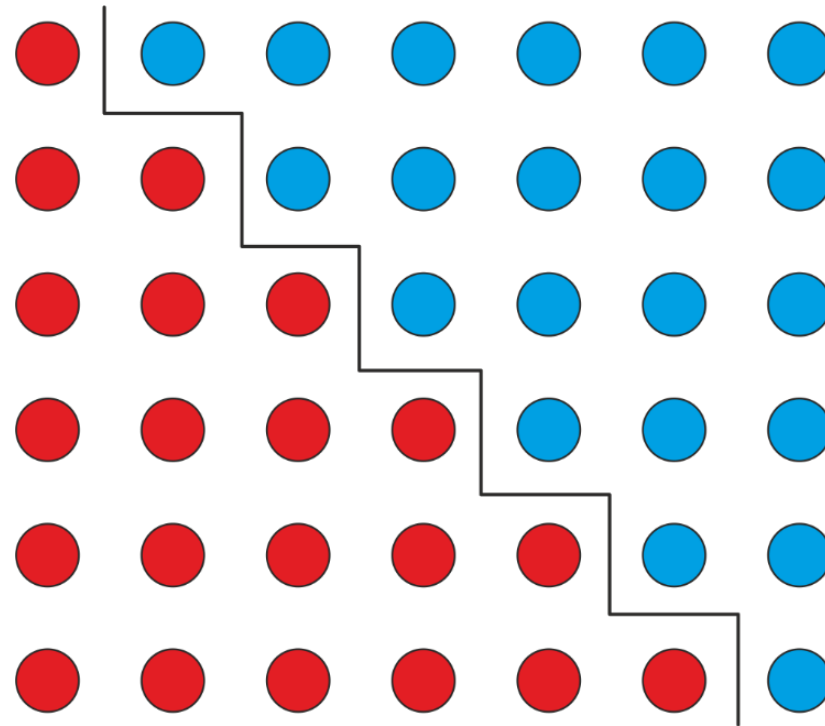




Liczba prostokątna 8×9

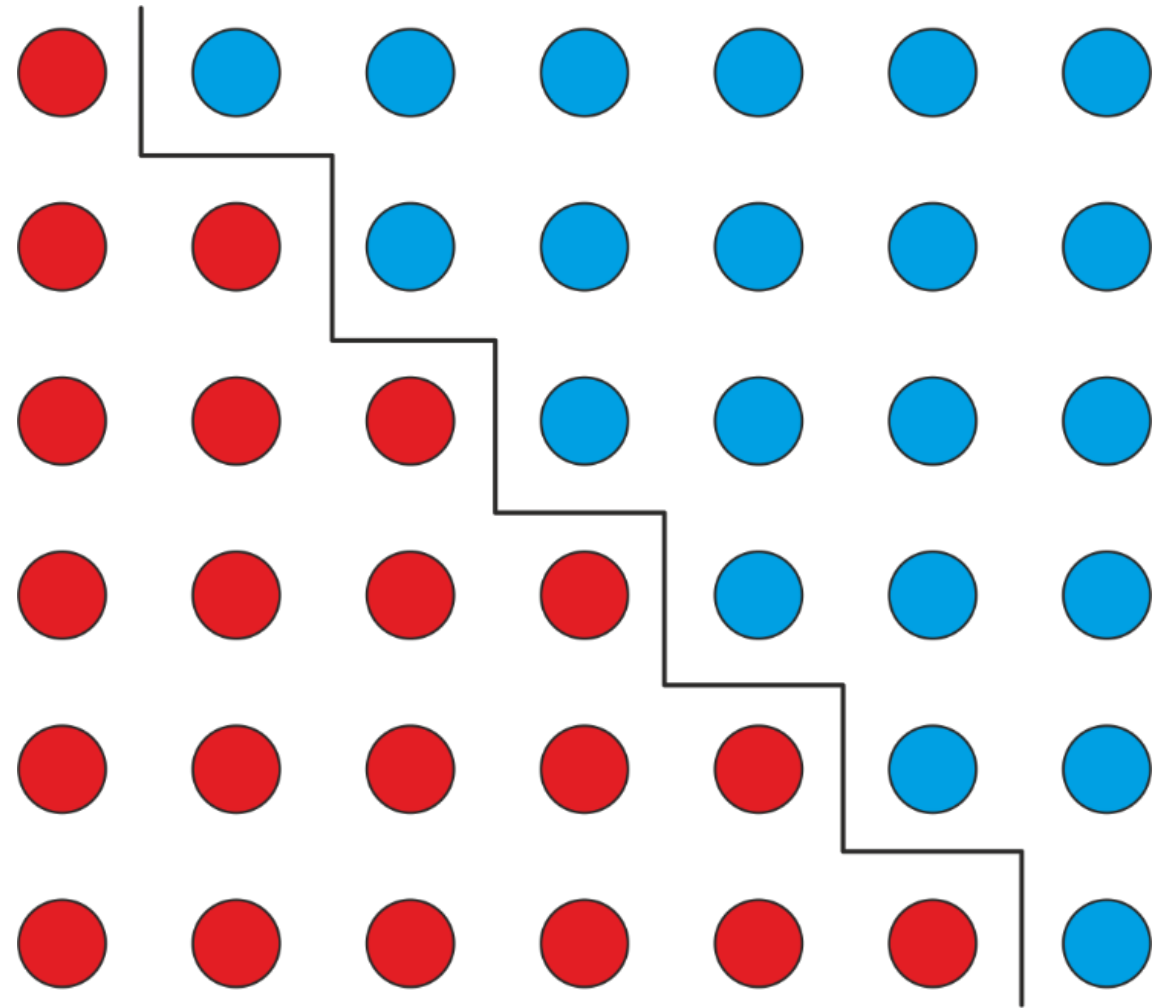


Suma kolejnych liczb naturalnych

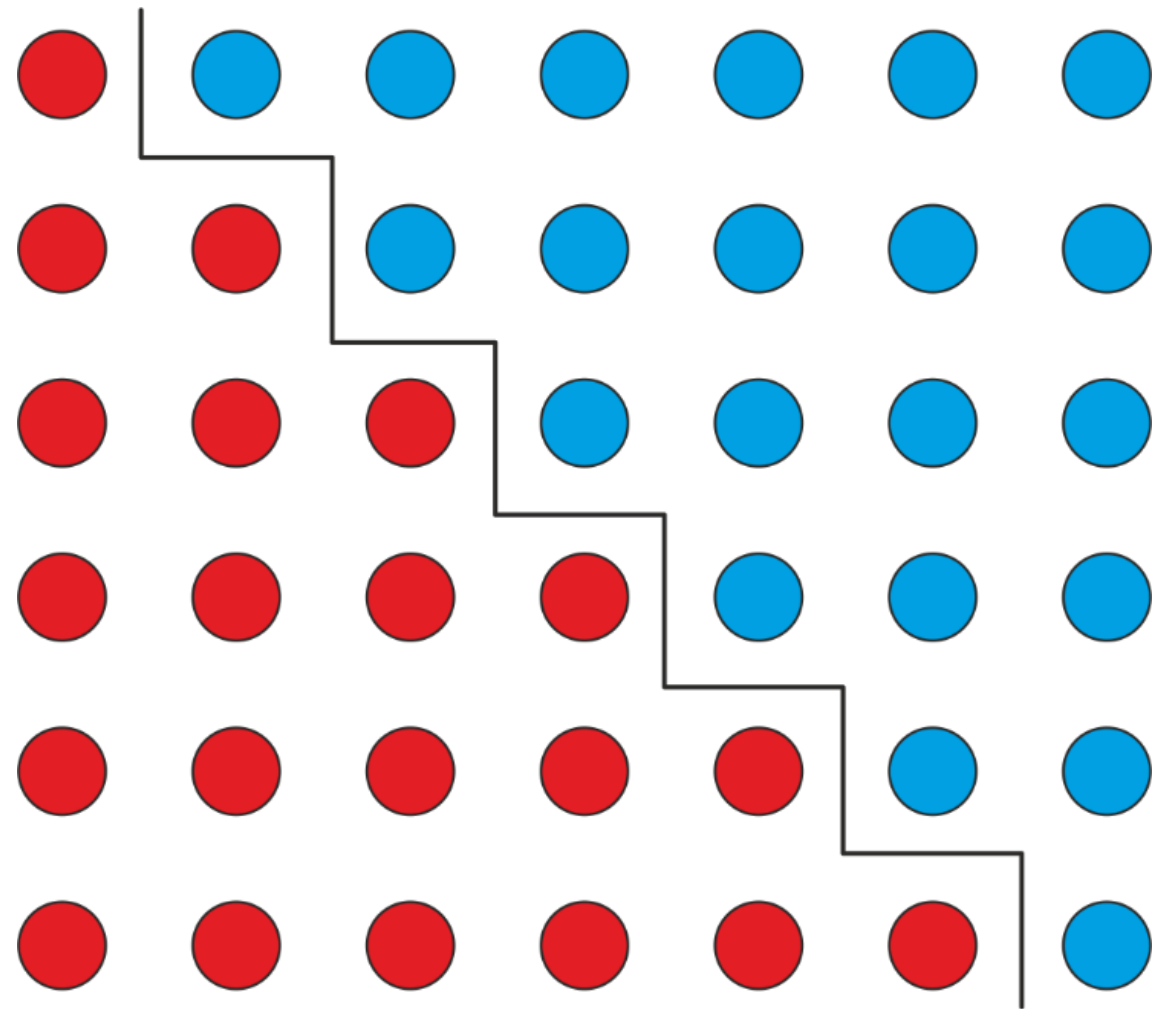


$$1 + 2 + \dots + n = \frac{1}{2}n(n + 1)$$

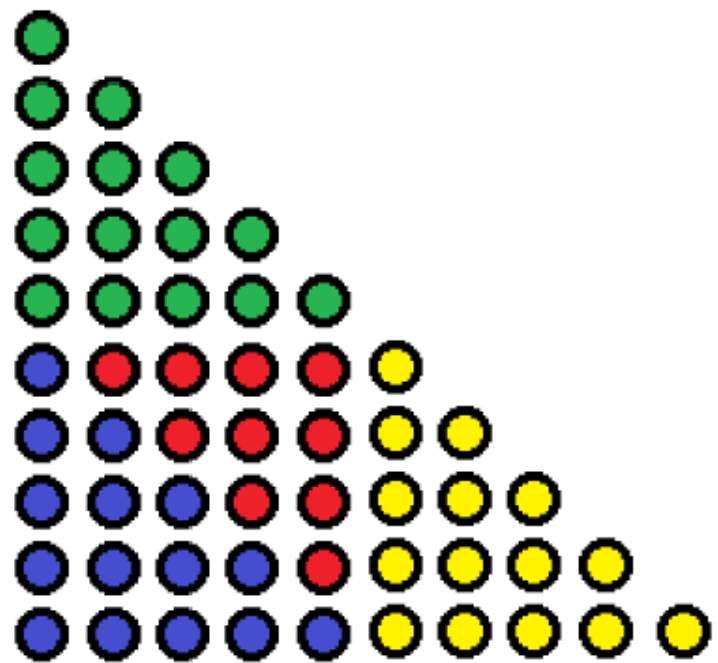
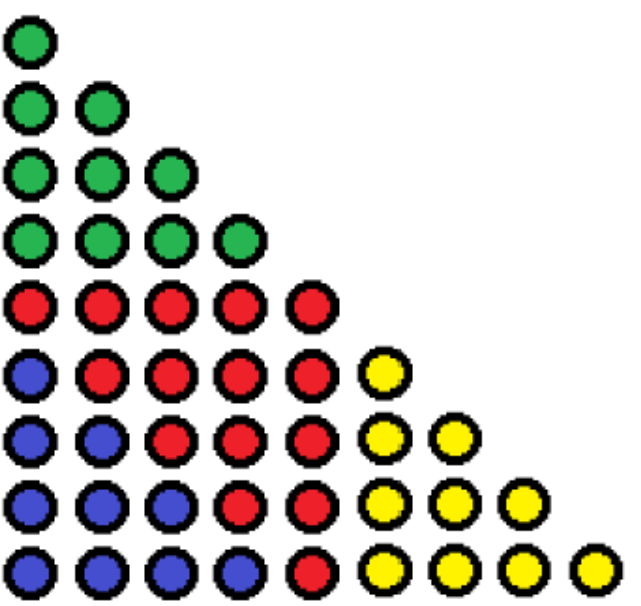
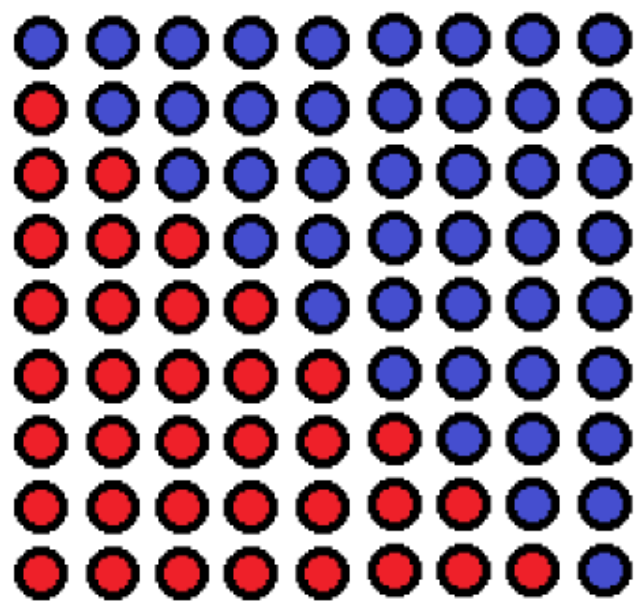
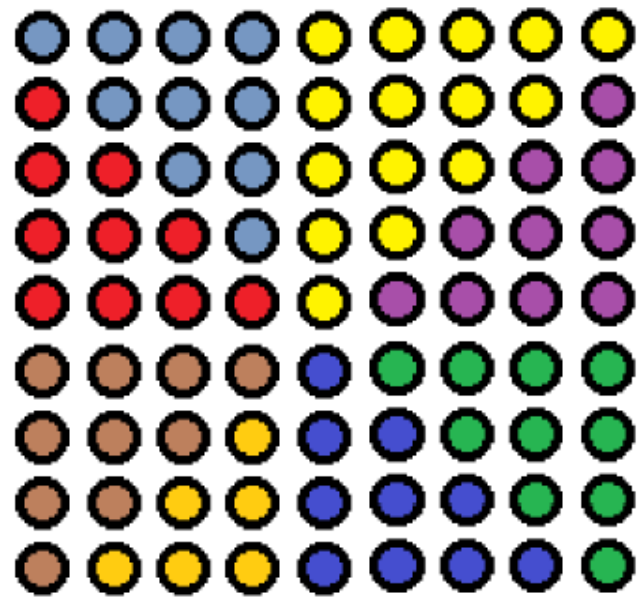
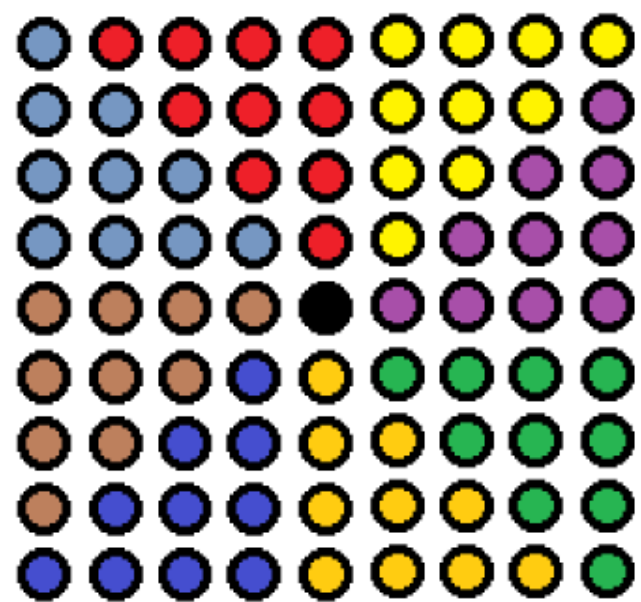
(Starożytni Grecy)



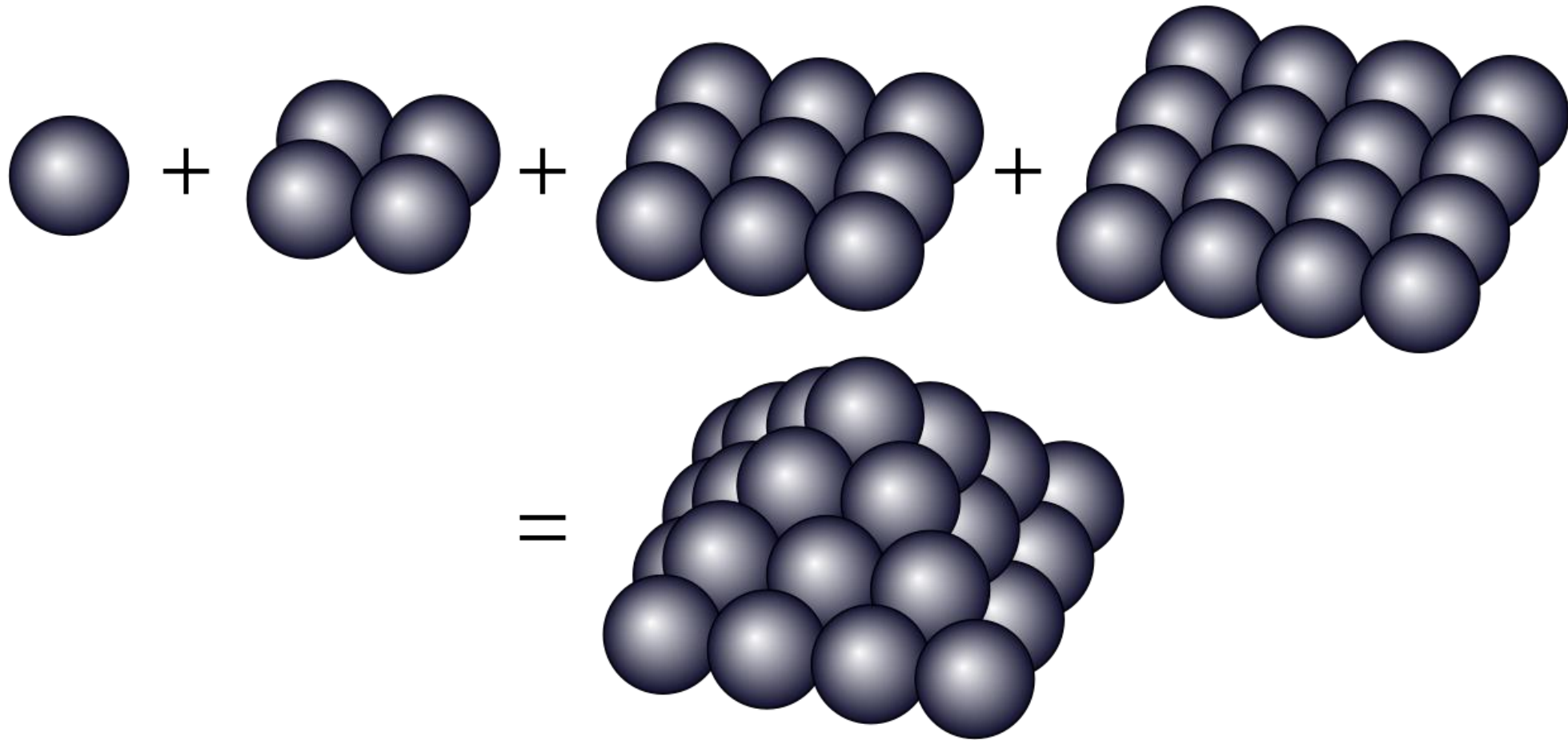
Suma kolejnych liczb parzystych

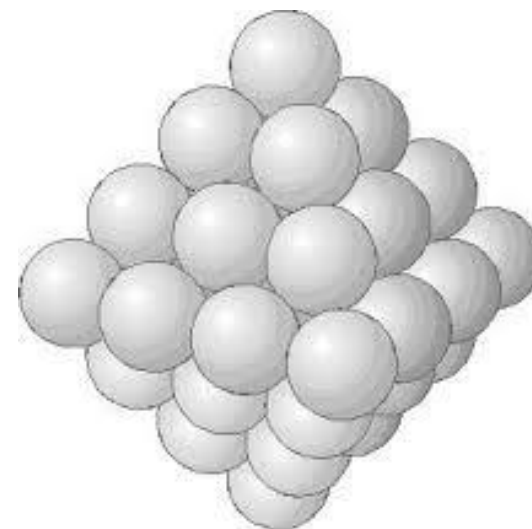
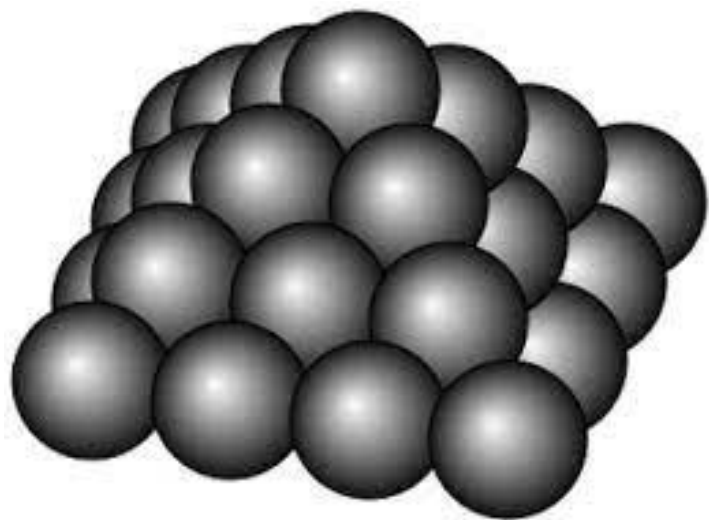


$$2 + 4 + 6 + \dots + 2024 = ?$$



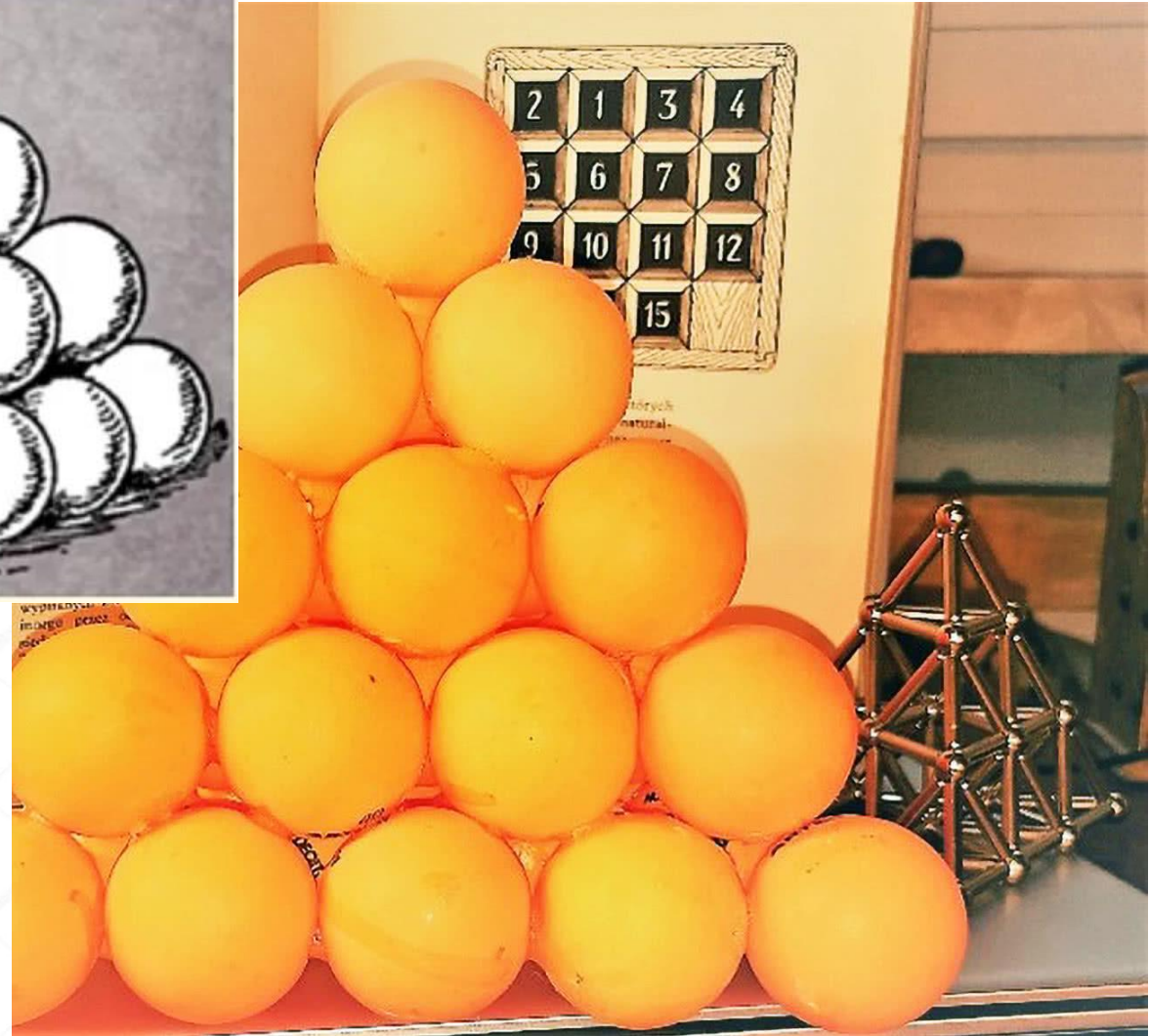
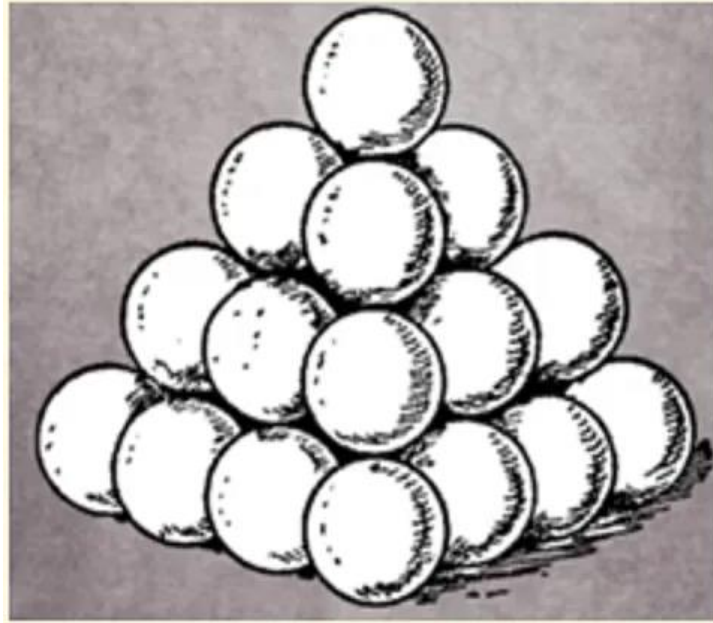
Suma liczb kwadratowych = liczba piramidalna





Suma dwóch kolejnych liczb piramidalnych =
liczba ośmiościenna

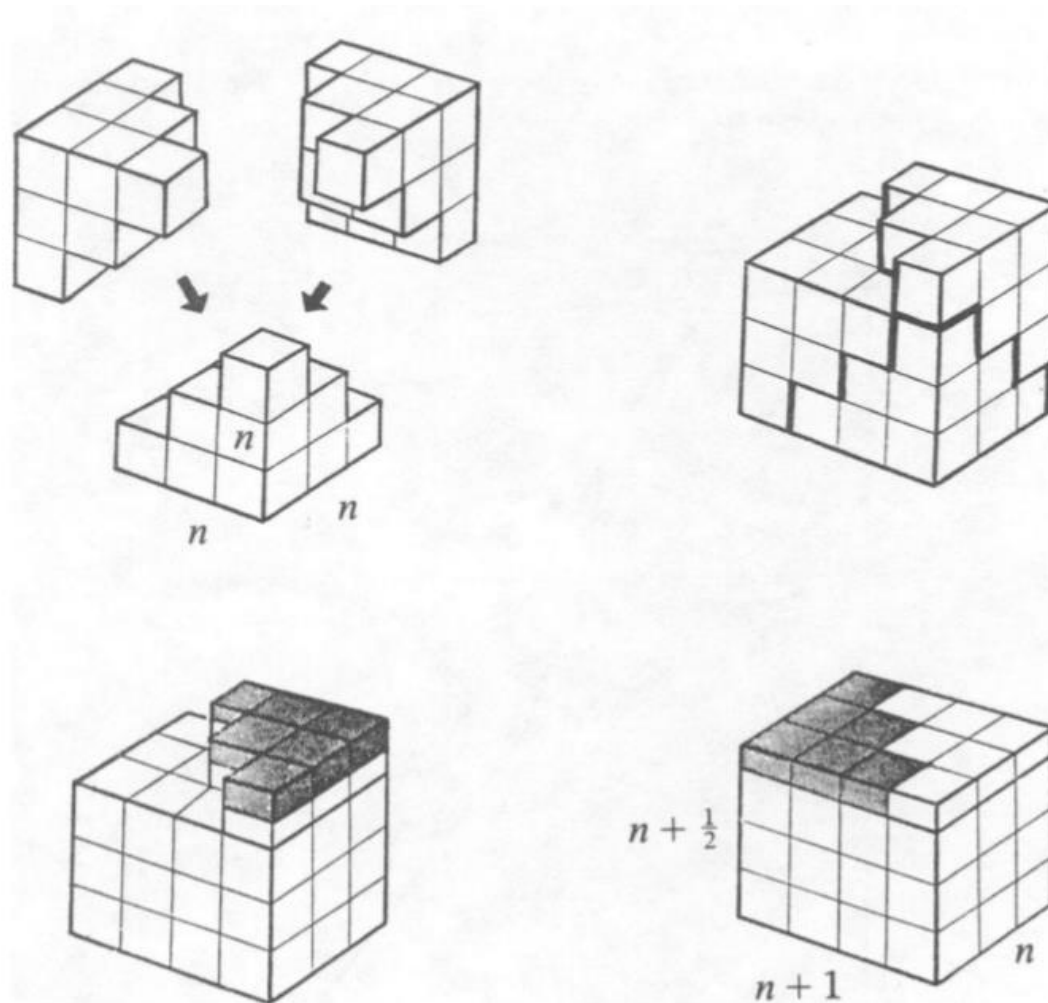
Suma liczb trójkątnych = liczba czworościenna



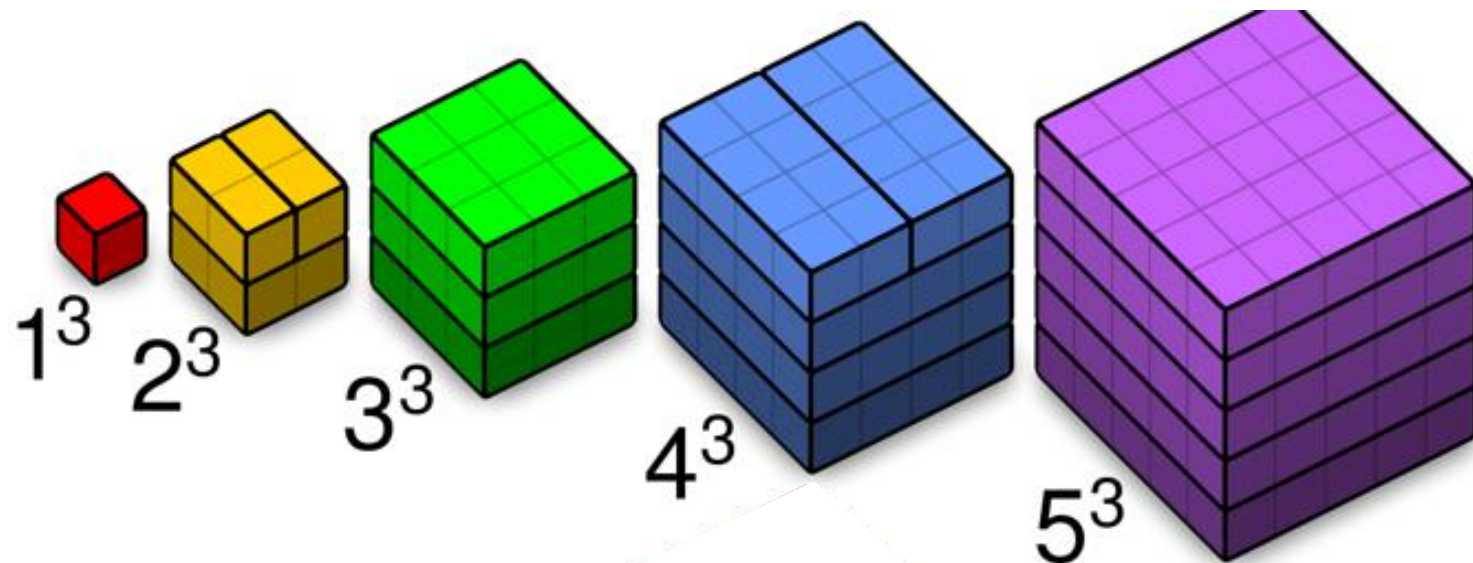
$$1^2 + 2^2 + 3^2 + \dots + 17^2 = ?$$

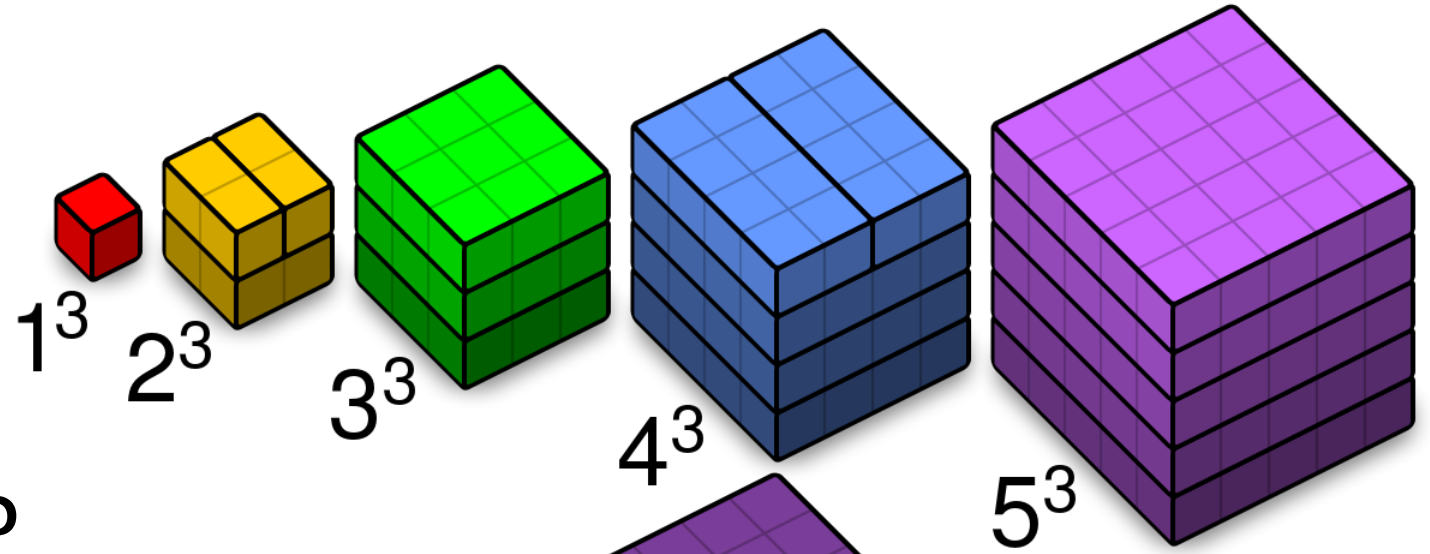
$$1^2 + 2^2 + \dots + n^2 = \frac{1}{3}n(n+1)(n+\frac{1}{2})$$

Suma kwadratów
kolejnych liczb
naturalnych



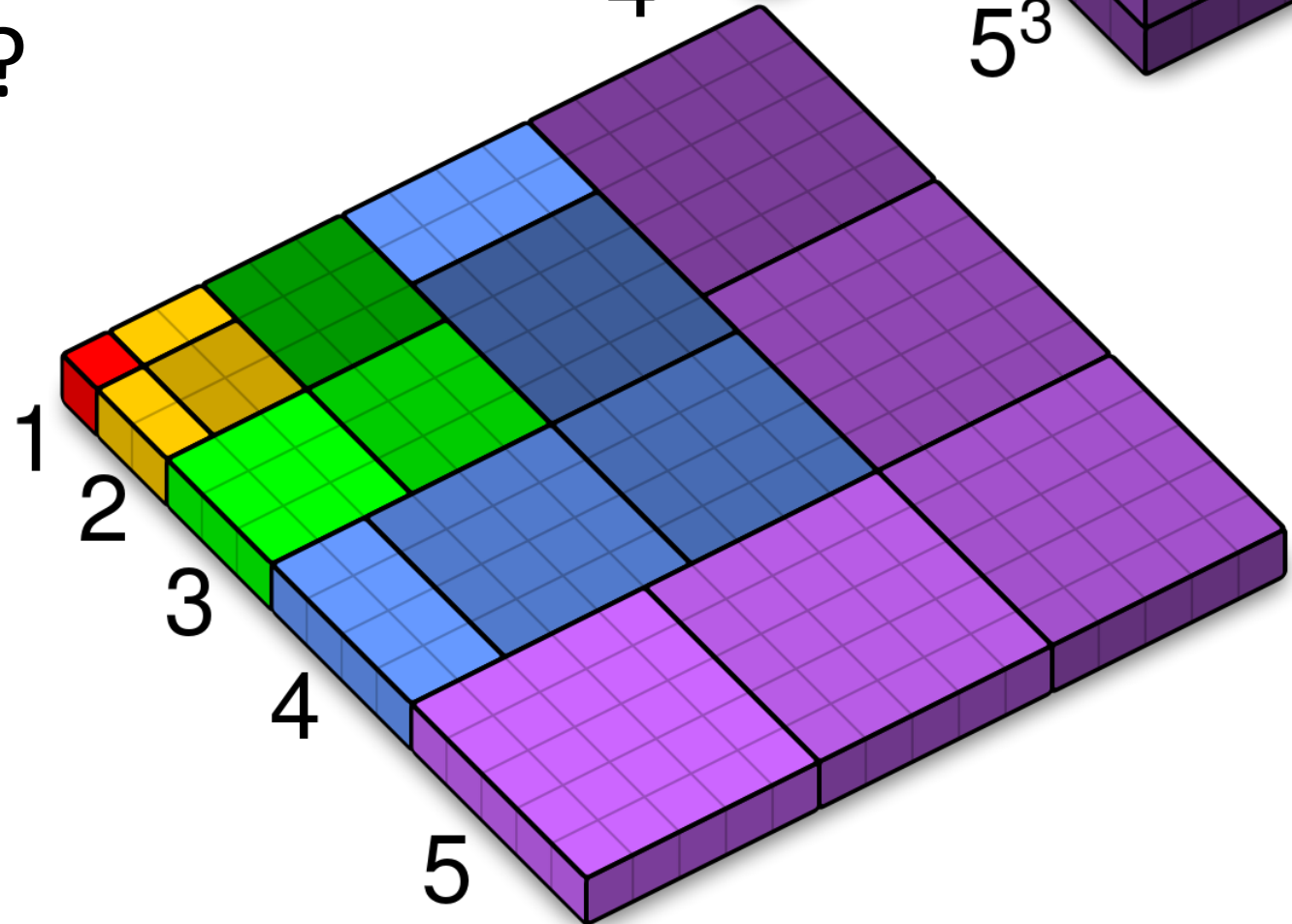
Liczby sześciennie



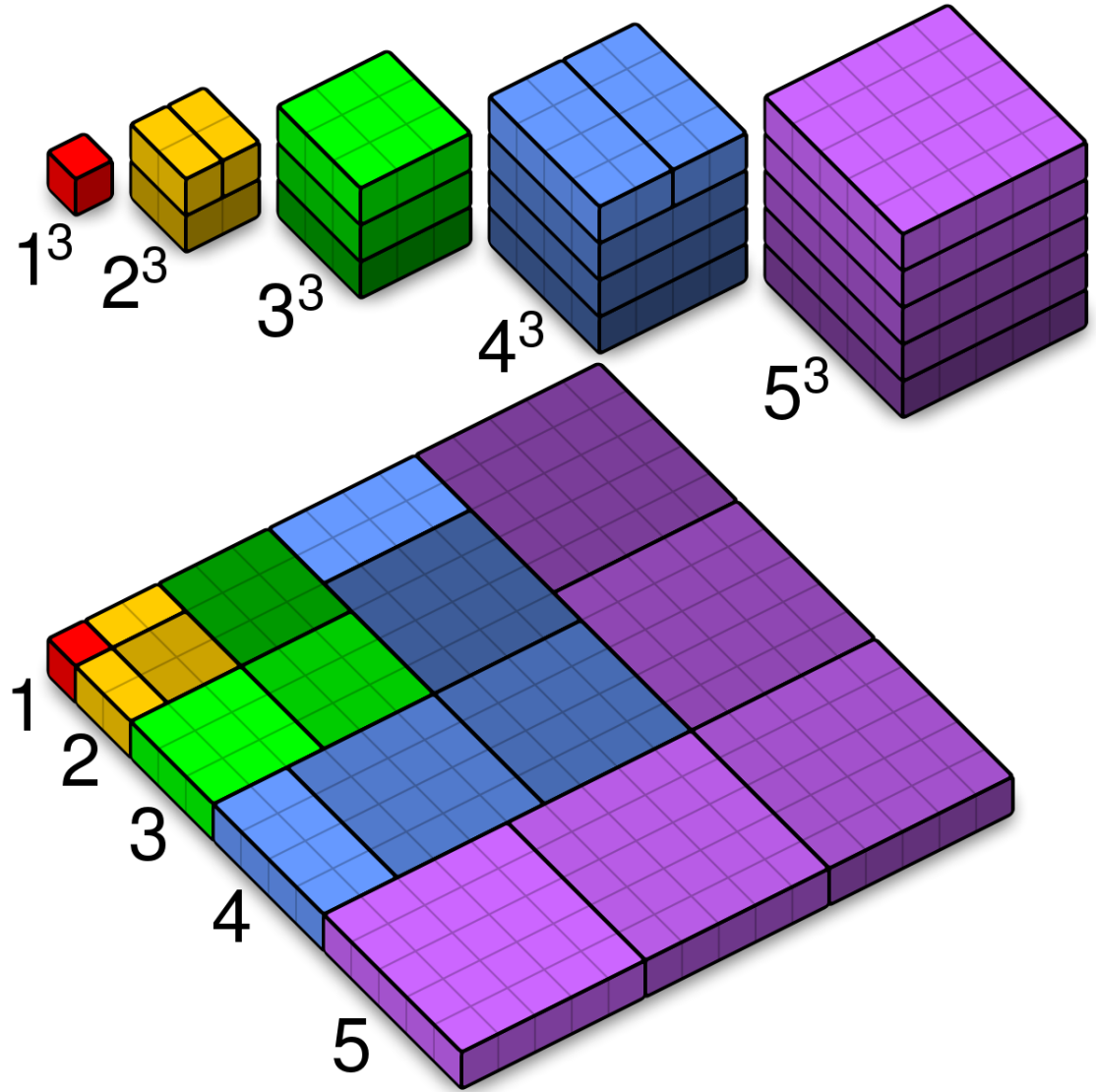


$$1^3 + 2^3 + 3^3 + \dots + 17^3 = ?$$

Suma sześciątów
kolejnych liczb
naturalnych

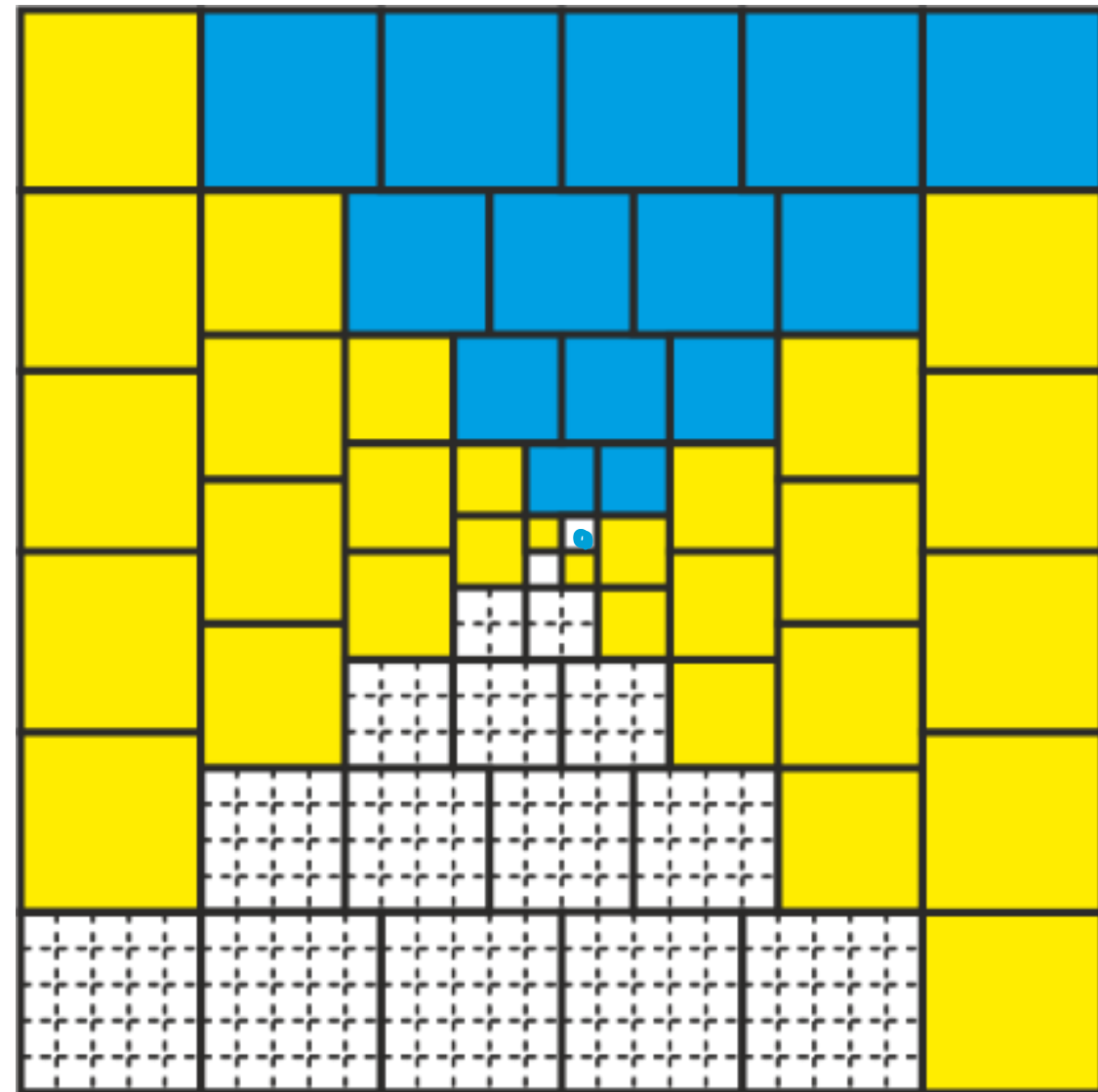


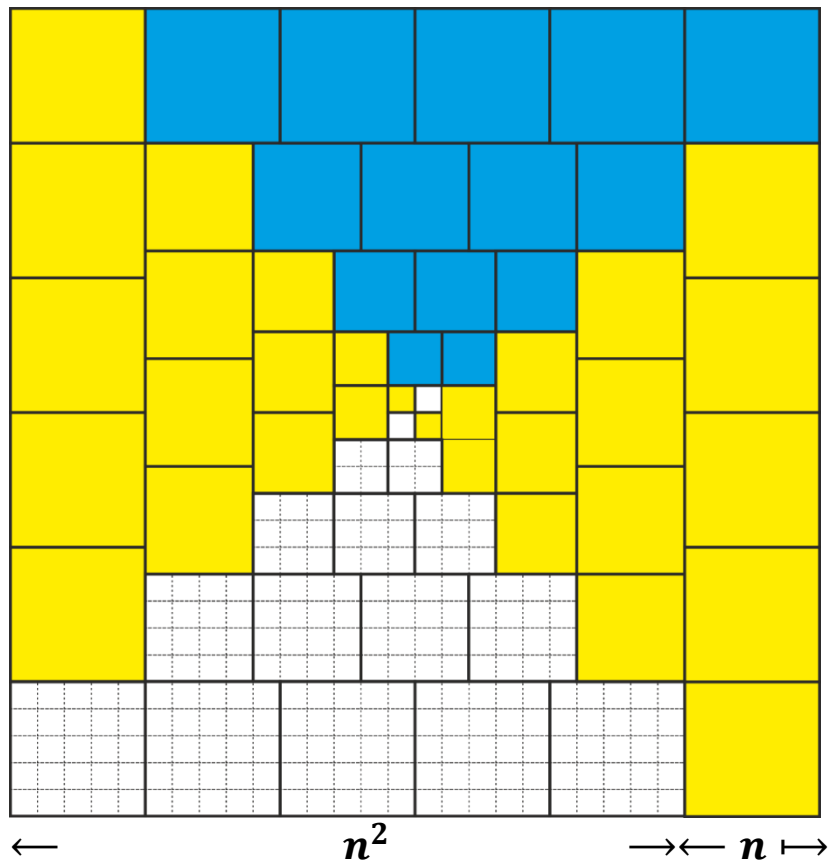
Suma sześciątów
kolejnych liczb
naturalnych



$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{1}{4} [n(n+1)]^2$$

Suma sześciątów kolejnych liczb naturalnych



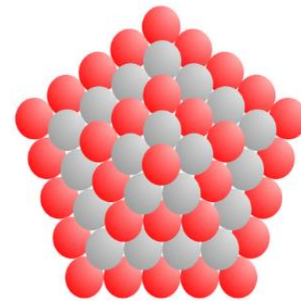
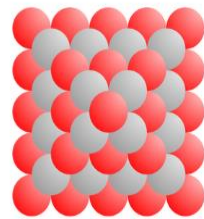
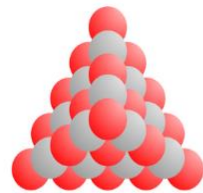


?

$$4 \cdot (1 \cdot 1^2 + 2 \cdot 2^2 + 3 \cdot 3^2 + \dots + 17 \cdot 17^2) = (n^2 + n)^2$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{1}{4} [n(n + 1)]^2$$

Liczby piramidalne (trójkątne, kwadratowe, pięciokątne)



Liczby
pentagonalne

$$n = 1$$

$$P_1 = 1$$

$$n = 2$$

$$P_2 = 1 + 5 = 6$$

$$n = 3$$

$$P_3 = 1 + 5 + 10 = 16$$

$$n = 4$$

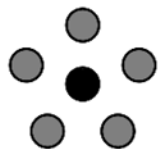
$$P_4 = 1 + 5 + 10 + 15 = 31$$

$$n = 5$$

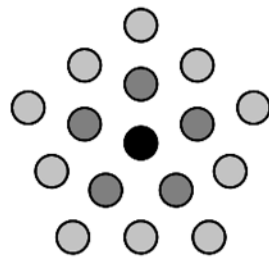
$$P_5 = 1 + 5 + 10 + 15 + 20 = 51$$



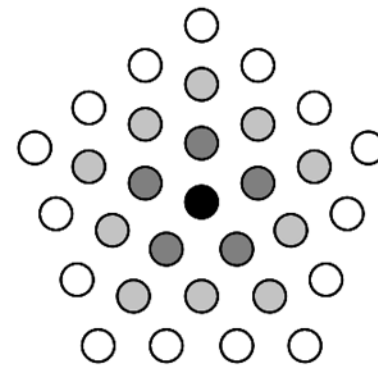
1



6



16



31

Liczby
heksagonalne

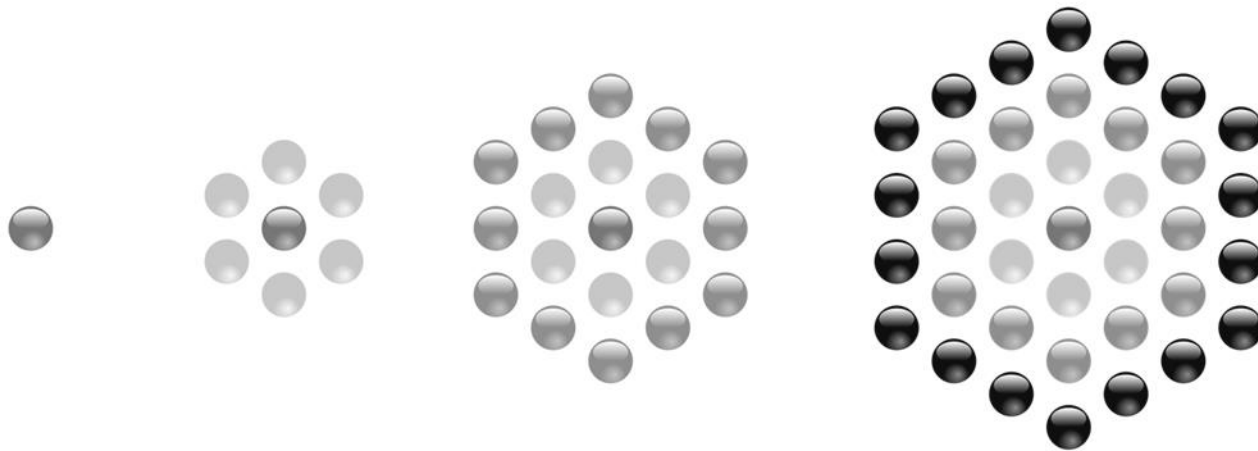
$$n = 1 \quad H_1 = 1$$

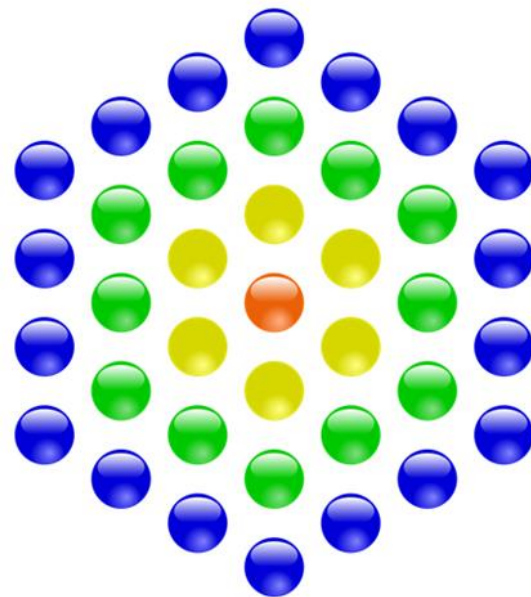
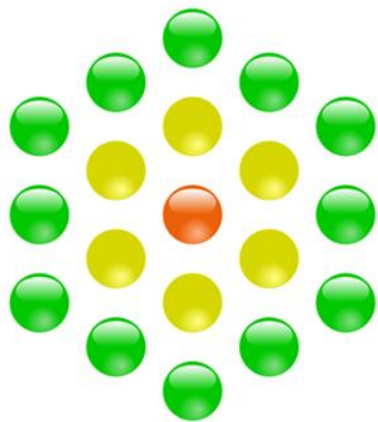
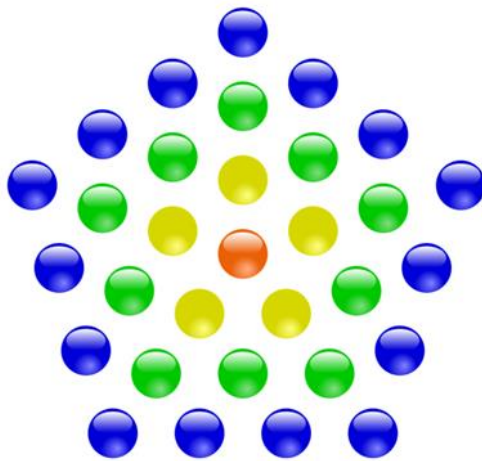
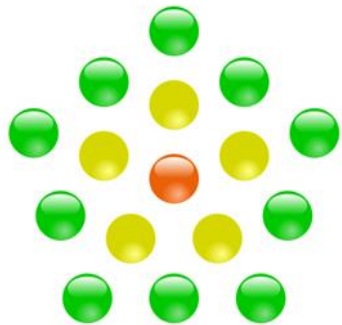
$$n = 2 \quad H_2 = 1 + 6 = 7$$

$$n = 3 \quad H_3 = 1 + 6 + 12 = 19$$

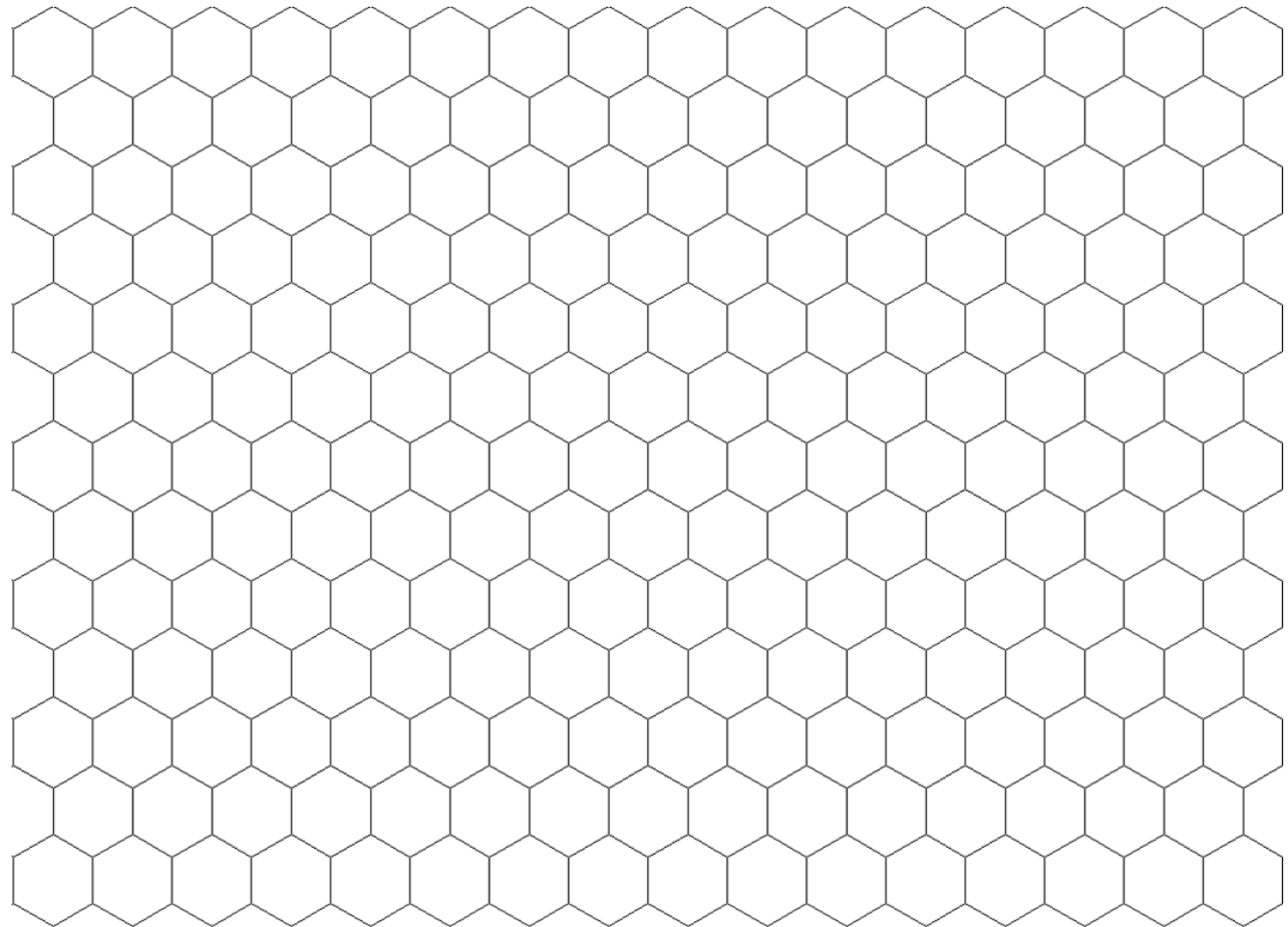
$$n = 4 \quad H_4 = 1 + 6 + 12 + 18 = 37$$

$$n = 5 \quad H_5 = 1 + 6 + 12 + 18 + 24 = 61$$

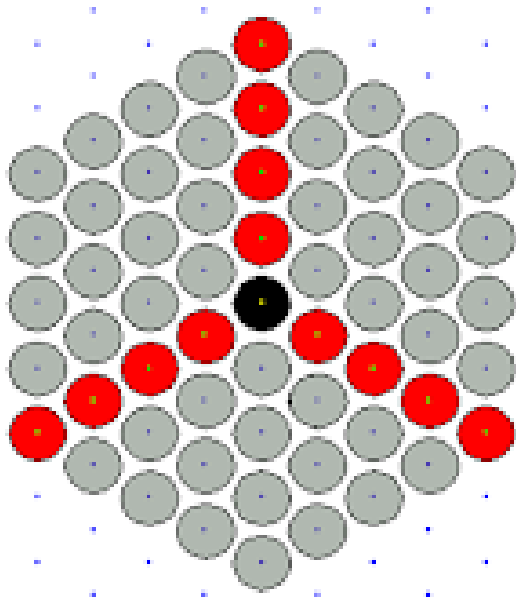




Liczby heksagonalne



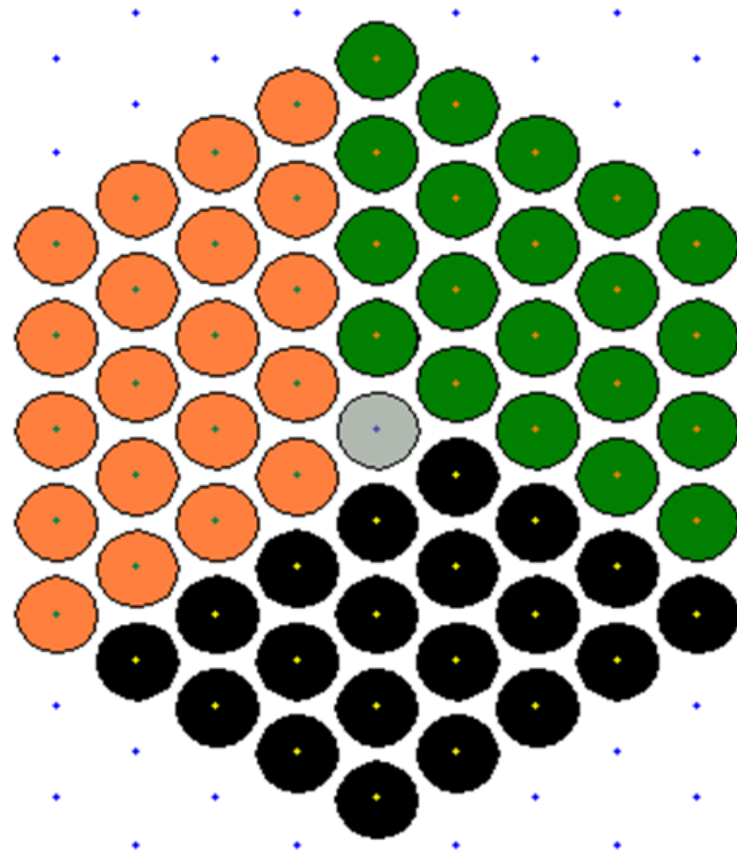
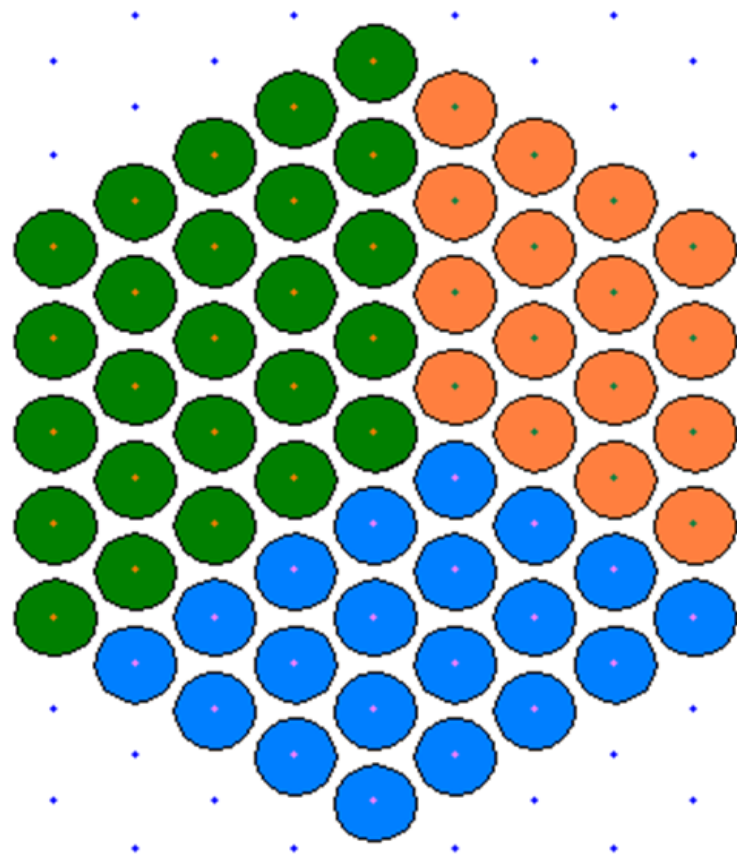
Liczby heksagonalne



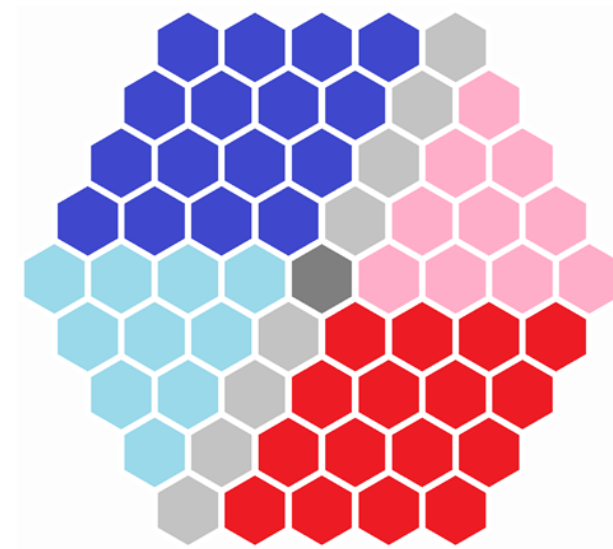
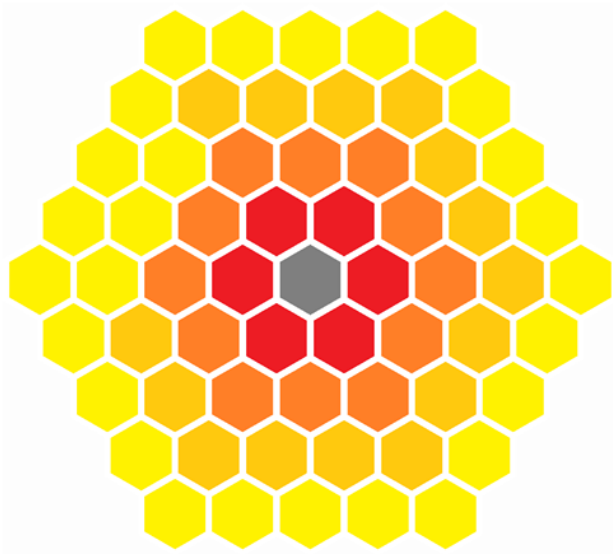
$n = 5$

$$3n^2 - 3n + 1$$

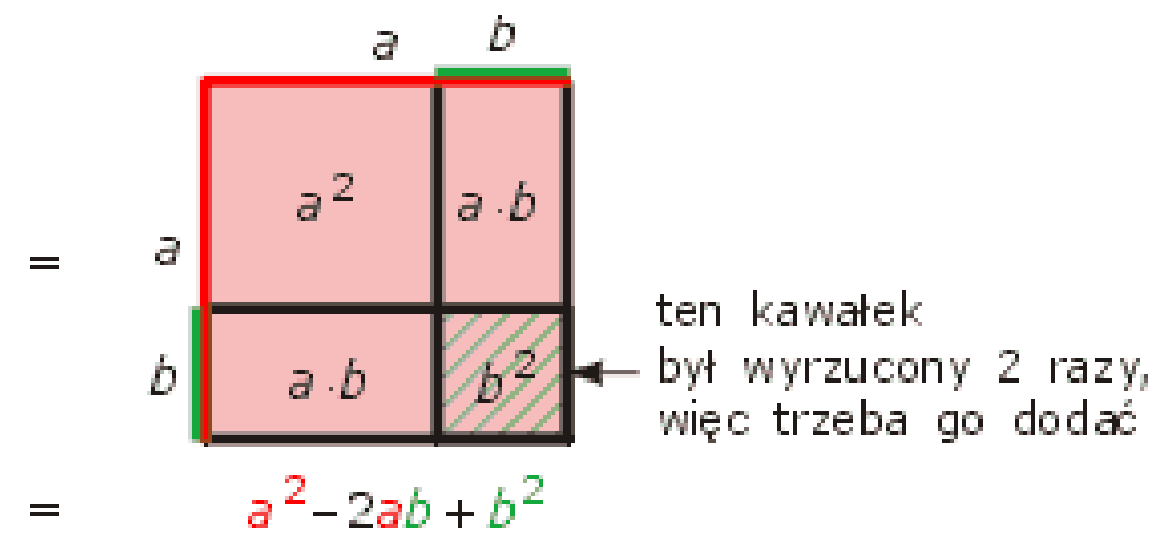
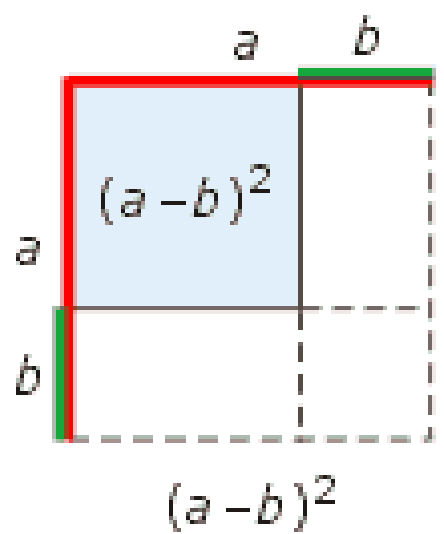
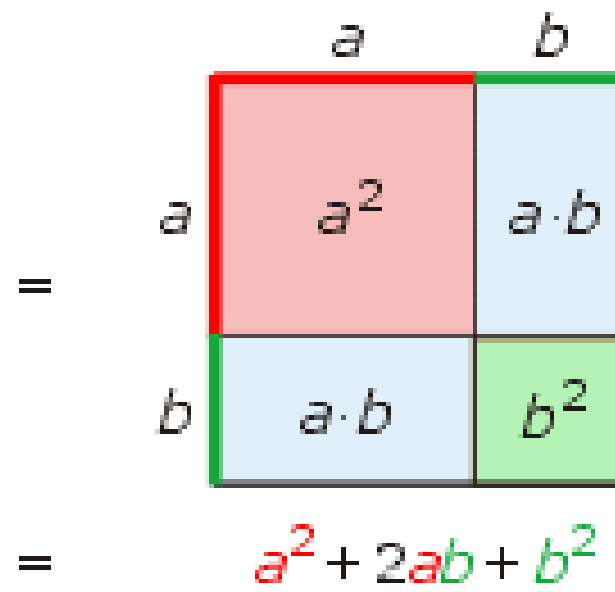
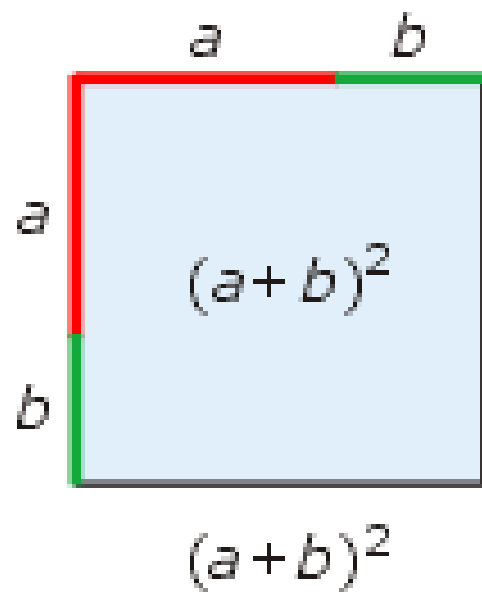
$n = 5$

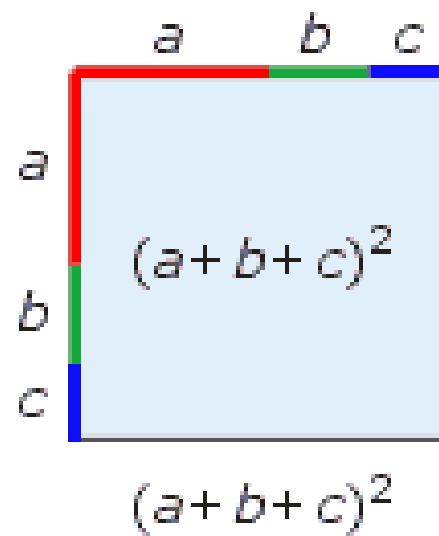




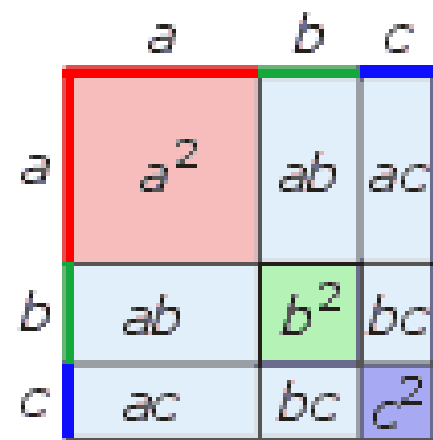


Rysunkowe wzory z algebry

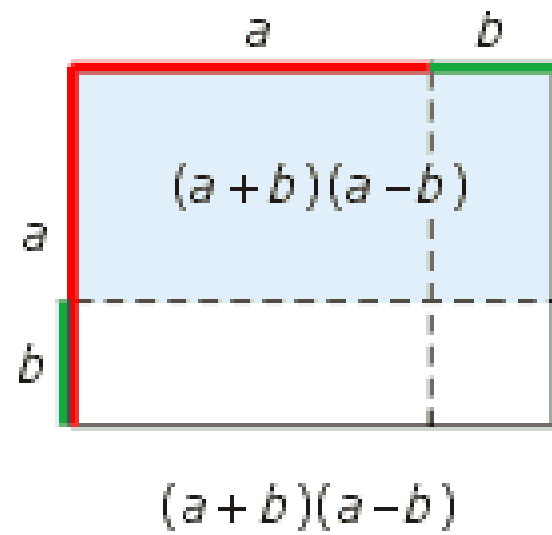




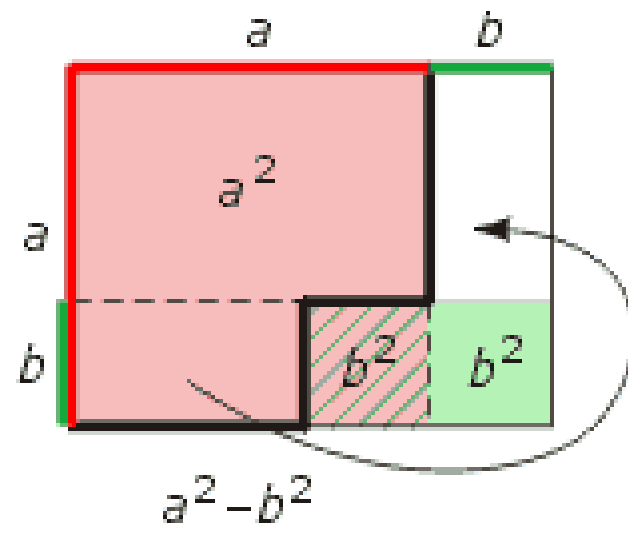
=



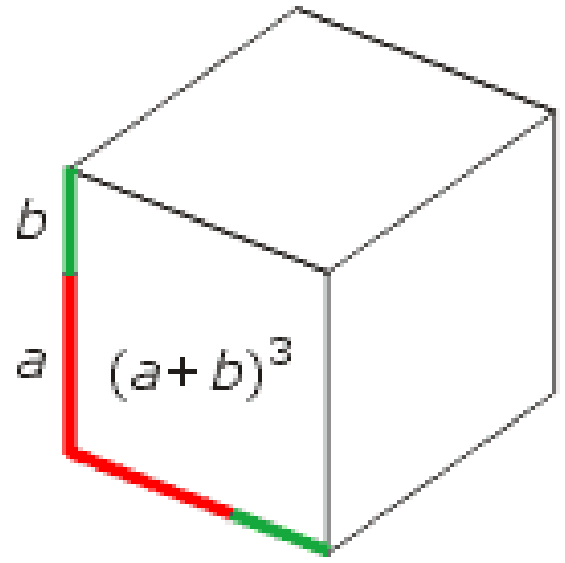
$= a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$



=

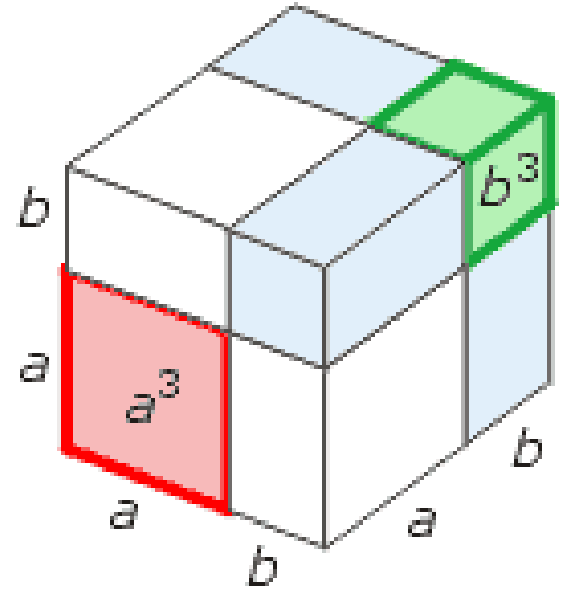


=



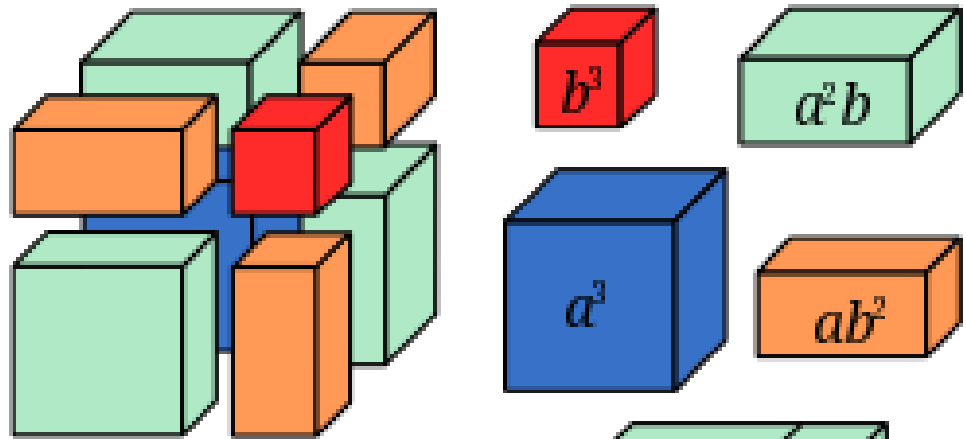
$$(a+b)^3$$

=



=

$$a^3 + b^3 + 3a^2b + 3b^2a$$



$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

