

Tower of Hanoi

PART A

Task 1 (group)

The Tower of Hanoi is a mathematical puzzle. The game consists of three pegs (or sticks), on one of which a variable number of discs of decreasing diameter are stacked to form a “tower”.

The aim of the game is to move the tower onto the last peg, according to the following rules:

- 1) only one disc can be moved at a time;
- 2) A disc can never be placed on top of one with a smaller diameter.

Try playing with the Hanoi tower with 3, 4 and 5 discs.

Suggestions: students can build their own Tower of Hanoi by cutting some discs out of cardboard. During the seminar you can play at <https://shorturl.at/yPR5v> (in Italian, use the Translate function on your browser).

An open mathematical problem concerns the solution of the Tower of Hanoi with any number of pegs and discs. Before studying the more complex case, however, it may be interesting to ask: what is the minimum number of moves required to solve the puzzle?

Task 2 (group)

After playing with the Hanoi tower with 3, 4 and 5 discs, complete the following table:

Number of discs (n)	Minimum number of moves (M_n)
1	1
2	
3	
4	
5	

What strategies did you use to solve the 3-disc tower in as few moves as possible?

What about the 4-disc tower? And the 5-disc tower? Explain your answer for each case.

Try to explain how to solve the tower for a generic number (n) of discs.

Task 3 (group)

Before playing with the 6-disc tower, try to guess the minimum number of moves you will use. Explain your reasoning.

Then try playing. Does your guess prove to be correct? Can you explain why it was correct or incorrect?

Outline for mathematical discussion:

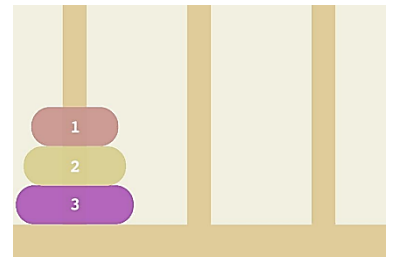
- To play, you always start by moving the smallest disc. However, depending on the number of discs you are playing with, you do not always move it to the same peg. Can you tell where it is best to move the smallest disc when playing with 3 discs? And with 4? And with 5? Are there any patterns between the number of discs and the peg where you start moving the first disc?
- What patterns can be observed between the number of discs and the number of moves?
- How can we express the minimum number of moves as a function of the number of discs in a formula?

PART B

Task 4 (group)

Try playing again, increasing the number of discs each time. This time, observe the movements of each disc. For convenience, you can number the discs as shown in the figure.

Can you tell which disc is moved the least?
And which one moves the most?



Task 5 (group)

In the following tables, blacken the box corresponding to the disc that is moved with each move.

3 discs								
	move number:							
Disc	1	2	3	4	5	6	7	Tot
1								
2								
3								

4 discs																
	move number:															
Disc	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Tot
1																
2																
3																
4																

5 discs																																
	move number:																															
Disc	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	Tot
1																																
2																																
3																																
4																																
5																																

What patterns do you observe?

How do you imagine the table describing the moves for the game with 6 discs?

For the teachers

- Discuss which knowledge, skills and competences are promoted by these tasks
- Specify the prior knowledge required.
- Discuss how to link this activity to the content and objectives of the Core Curriculum.
- What further ideas can be found for the discussion topic in PART B?

Further ideas

Tower of Hanoi, induction and recursive programming – <https://www.youtube.com/watch?v=rf6uf3jNjbo>

Tower of Hanoi and music – <https://www.youtube.com/watch?v=PGuRmqpr6Oo>