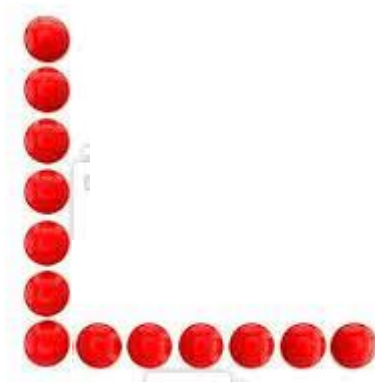
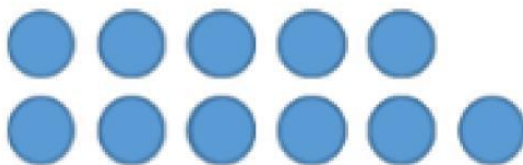
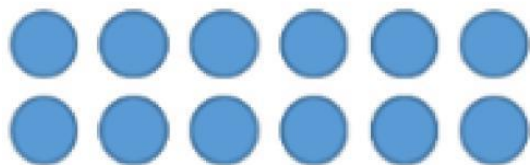
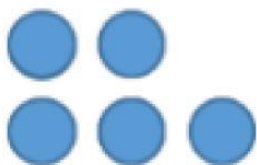
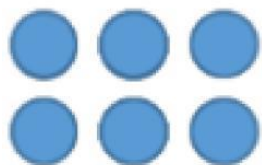


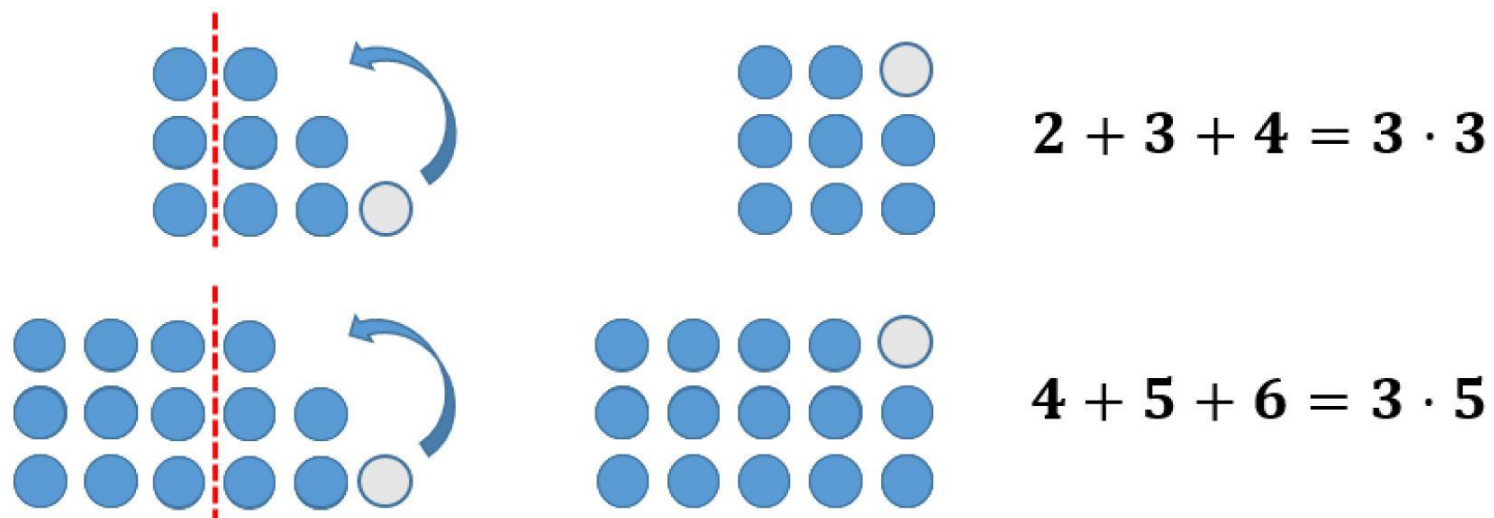
# ARYTMETYKA Z KULEK

# Liczby parzyste i nieparzyste



Parzystość sumy liczb

$$=3n+3=3\cdot(n+1)$$



suma trzech kolejnych liczb  
jest podzielna przez 3

$$n + (n+1) + (n+2) = 3n + 3 = 3(n+1)$$

# Liczby kwadratowe

1



4



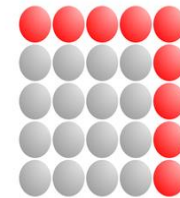
9



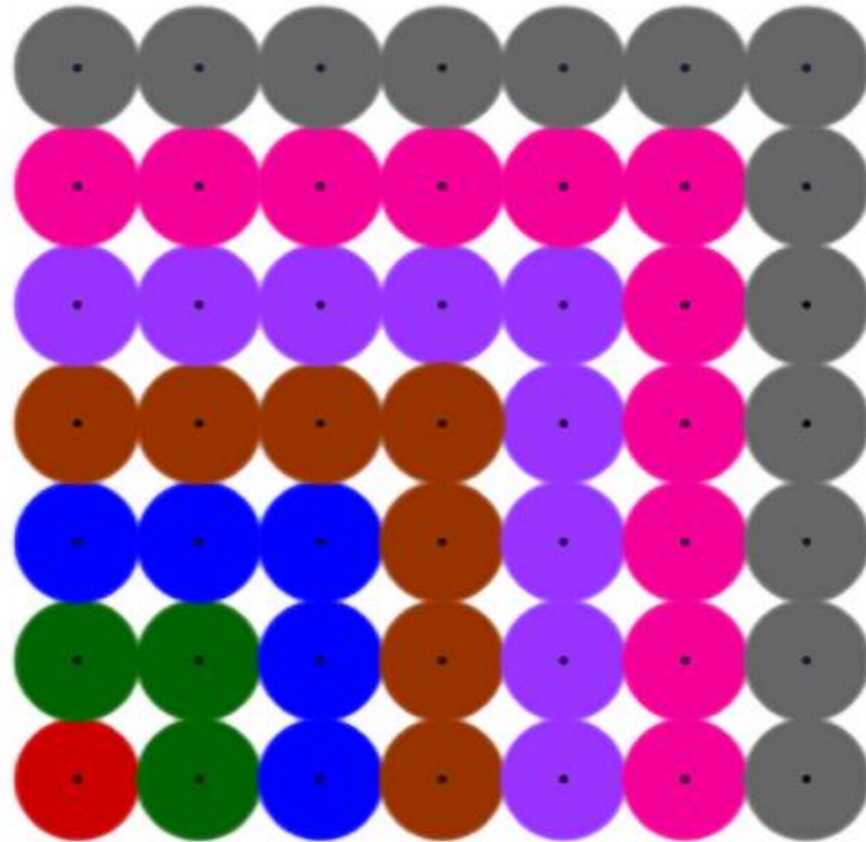
16



25



# Suma kolejnych liczb nieparzystych



$$1+3+5+\dots+117 = ?$$

# Liczby trójkątne

1



3



6

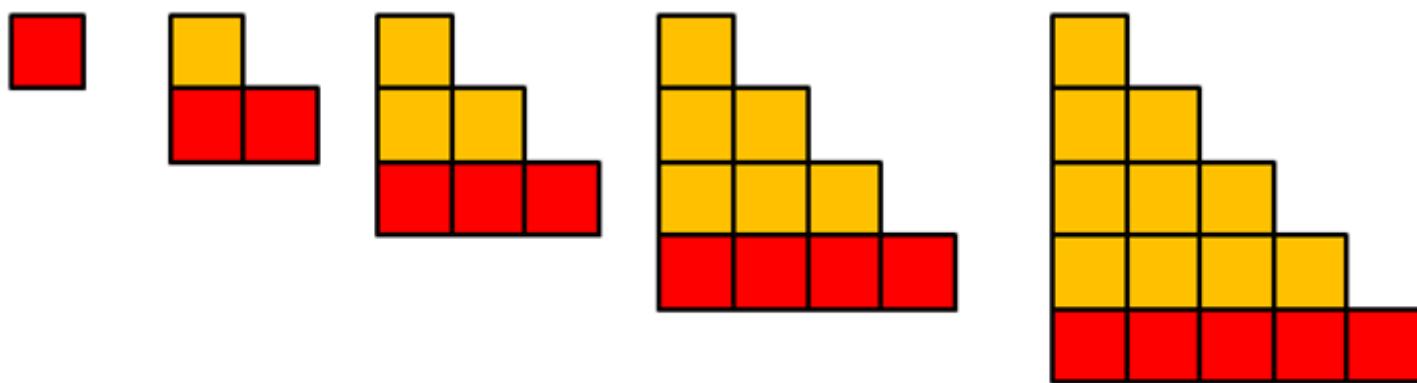
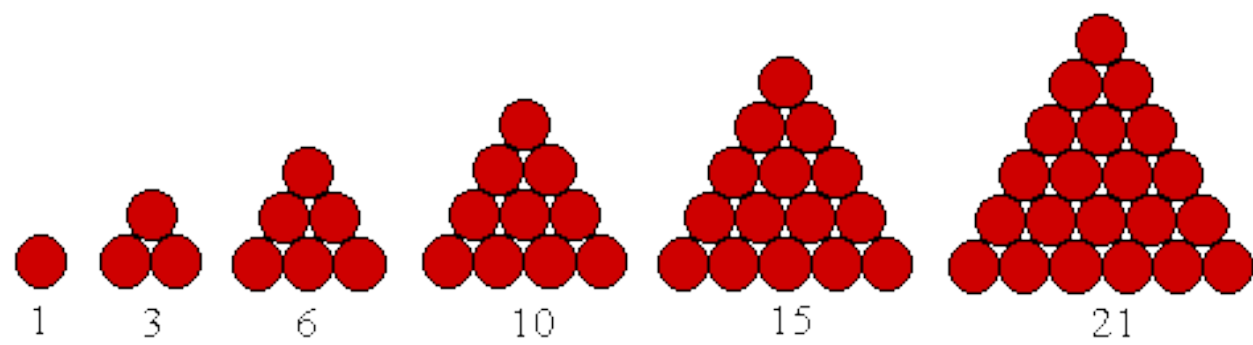


10



15



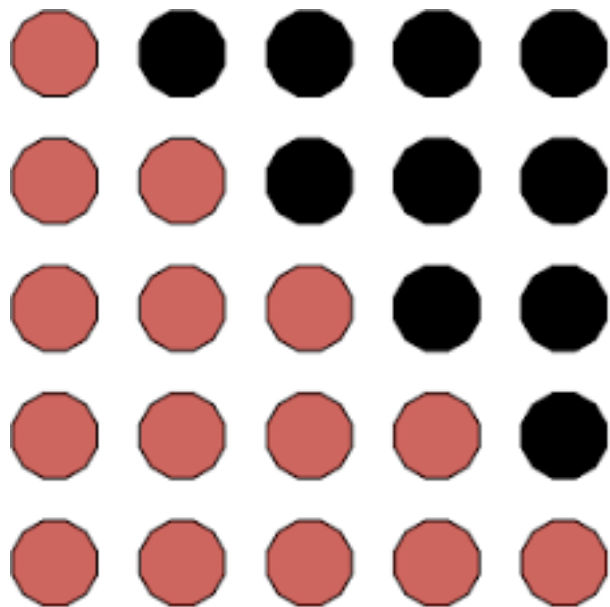




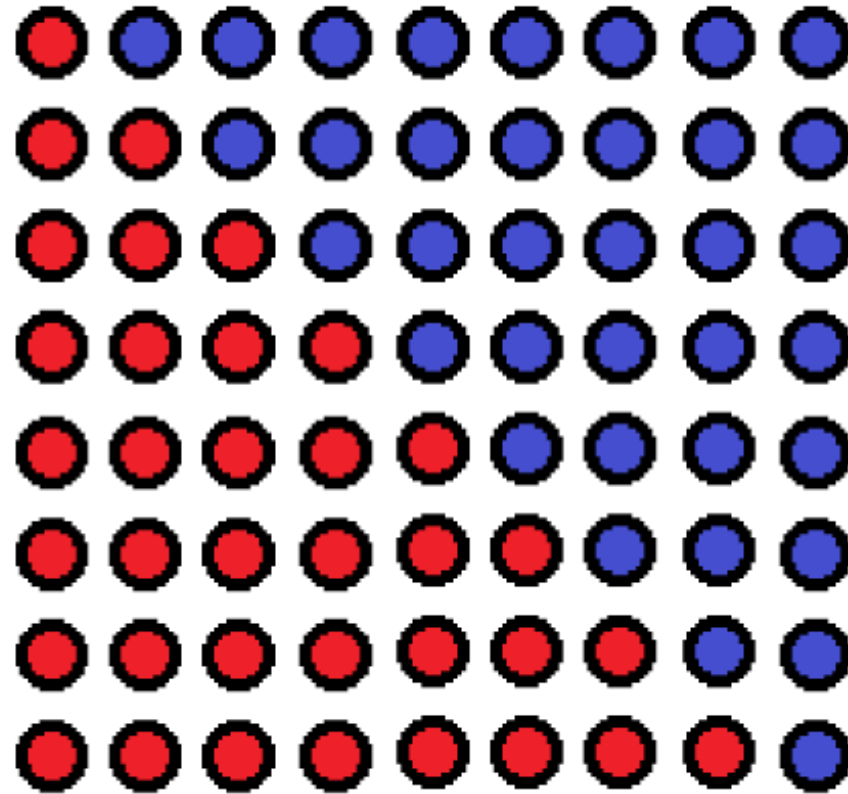




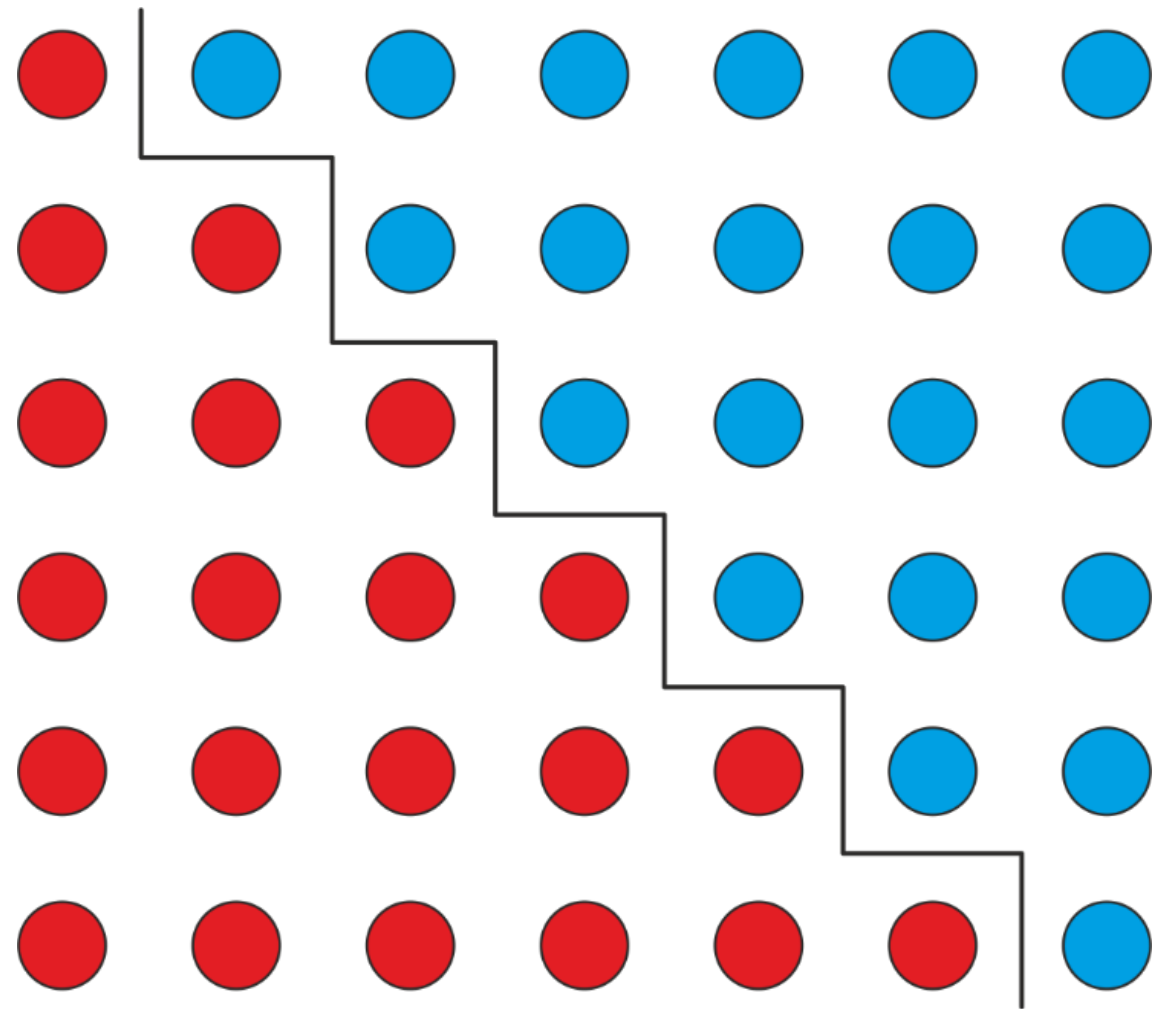
suma dwóch kolejnych liczb trójkątnych



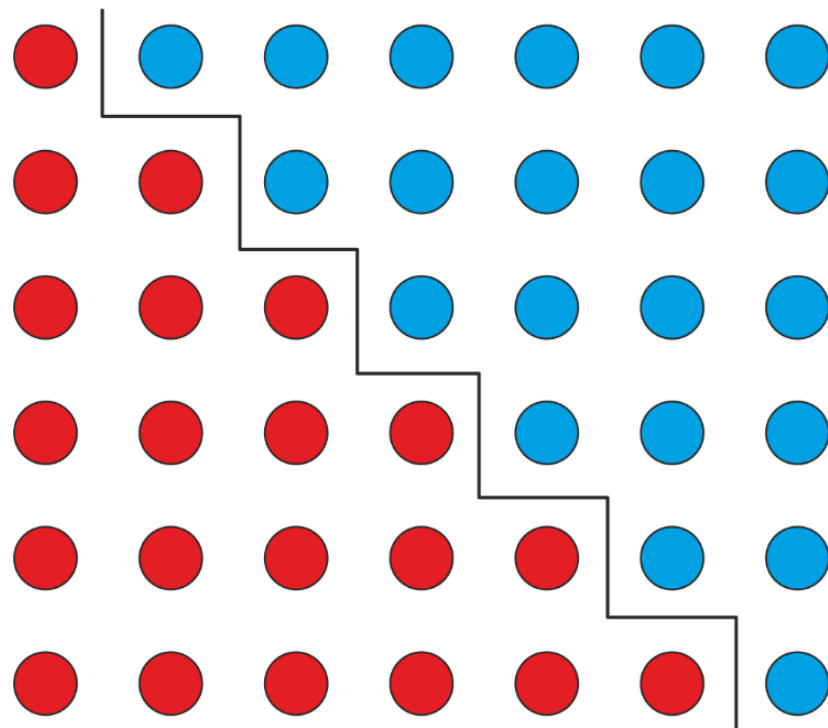
podwojenie liczby trójkątnej



liczba prostokątna 8×9

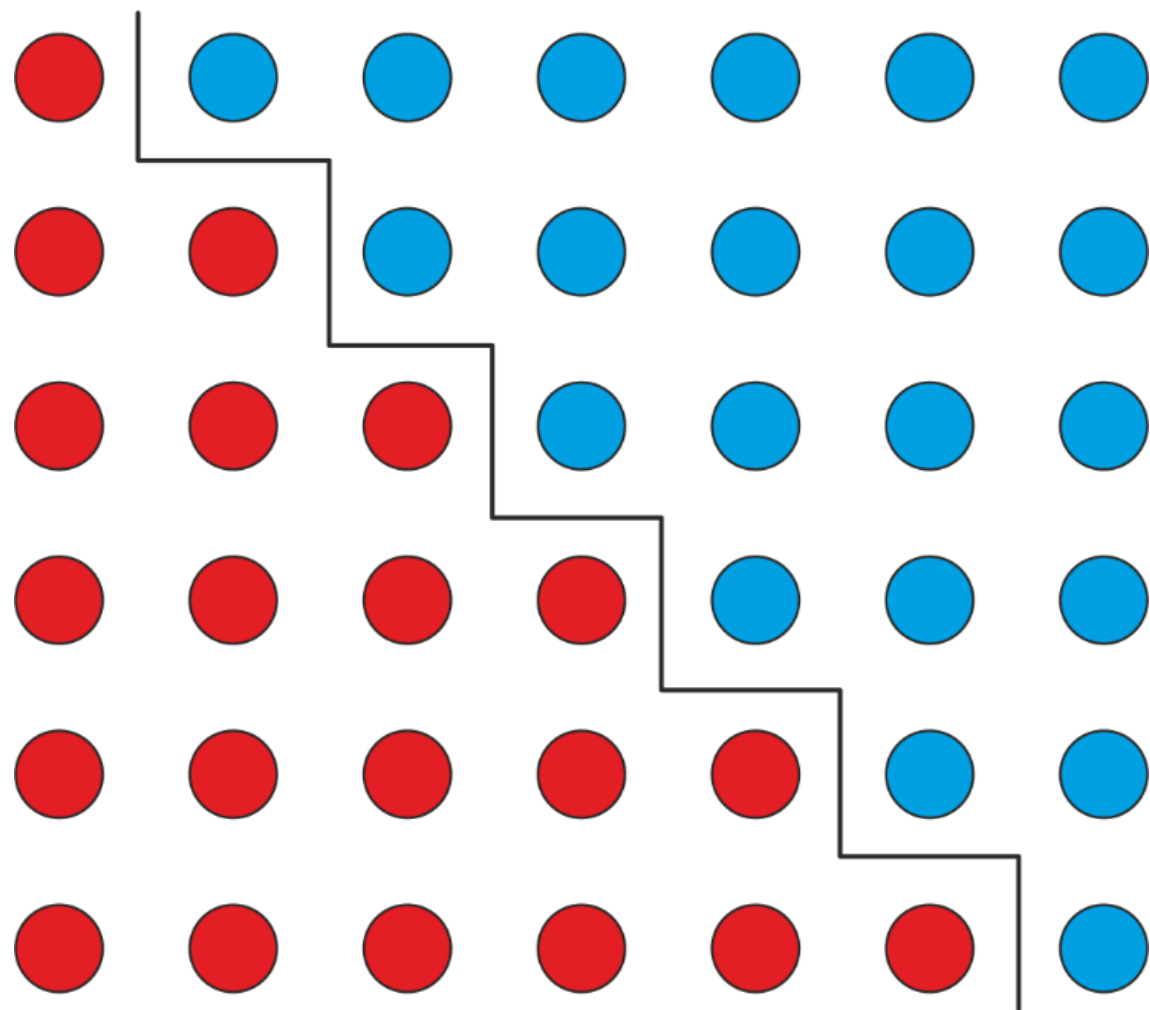


Suma kolejnych liczb naturalnych

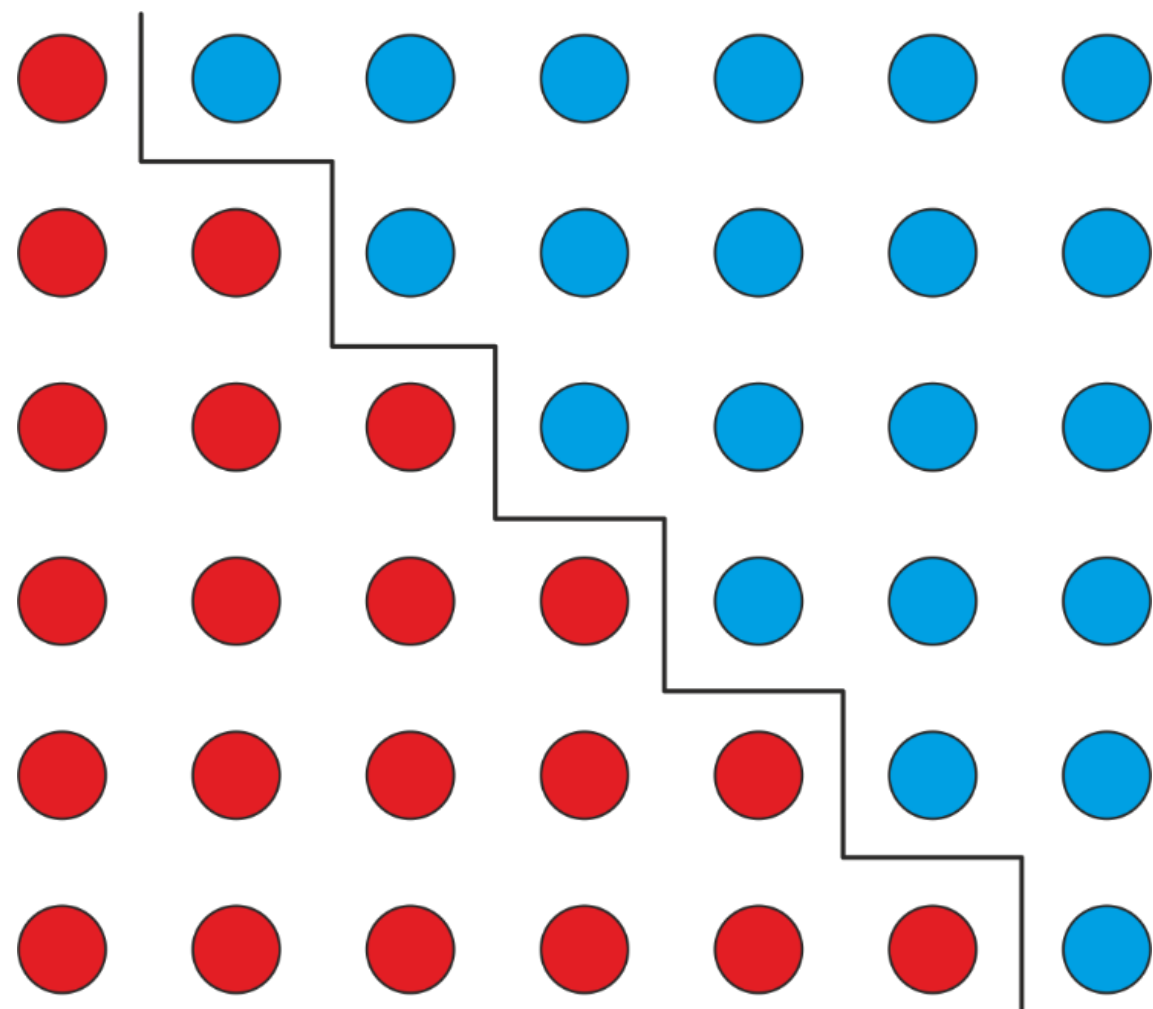


$$1 + 2 + \dots + n = \frac{1}{2}n(n + 1)$$

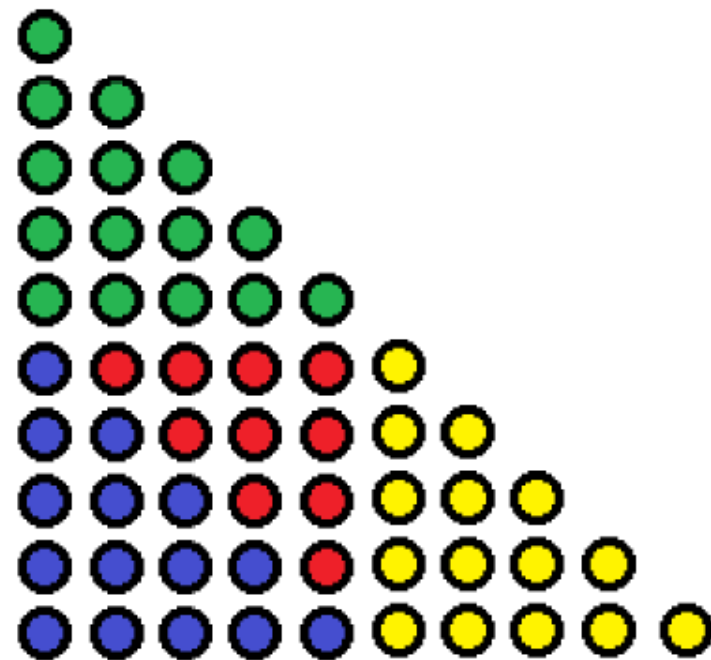
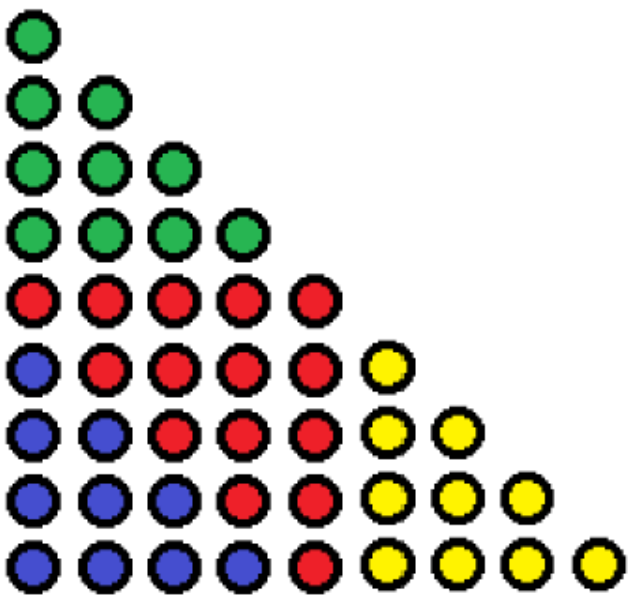
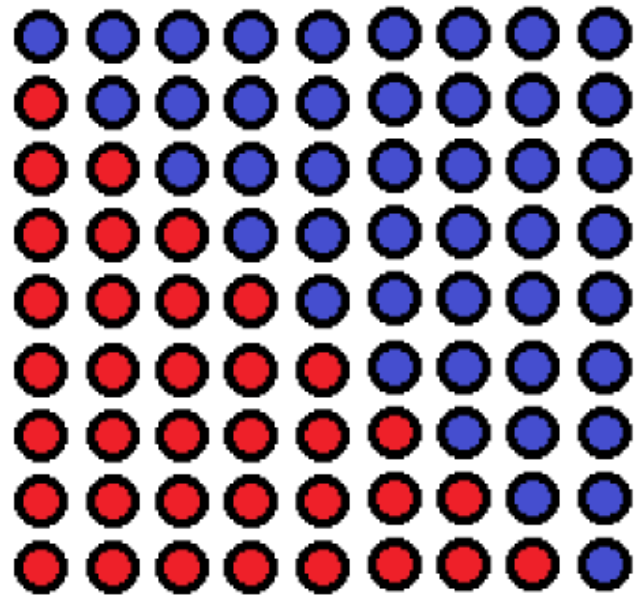
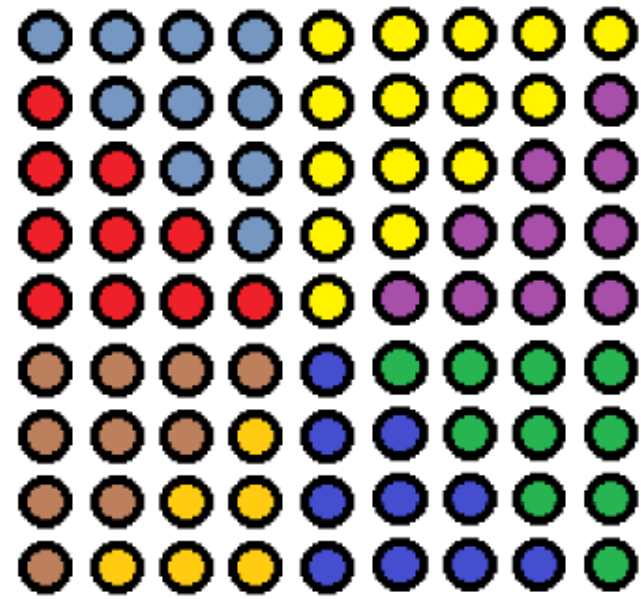
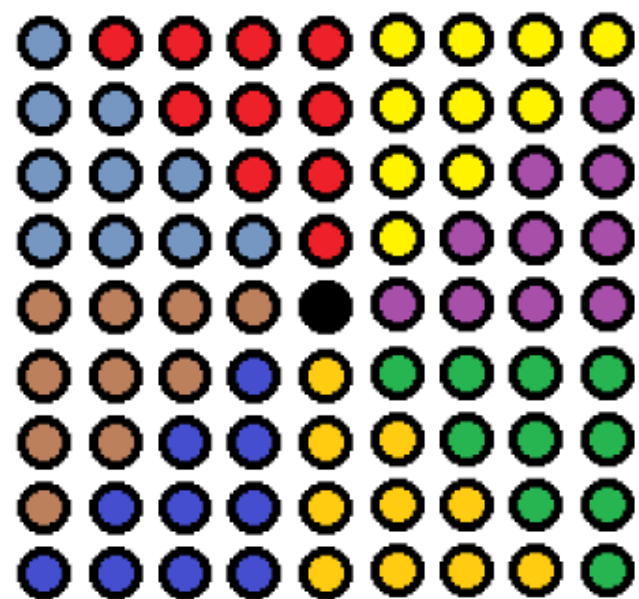
(Starożytni Grecy)



Suma kolejnych liczb parzystych

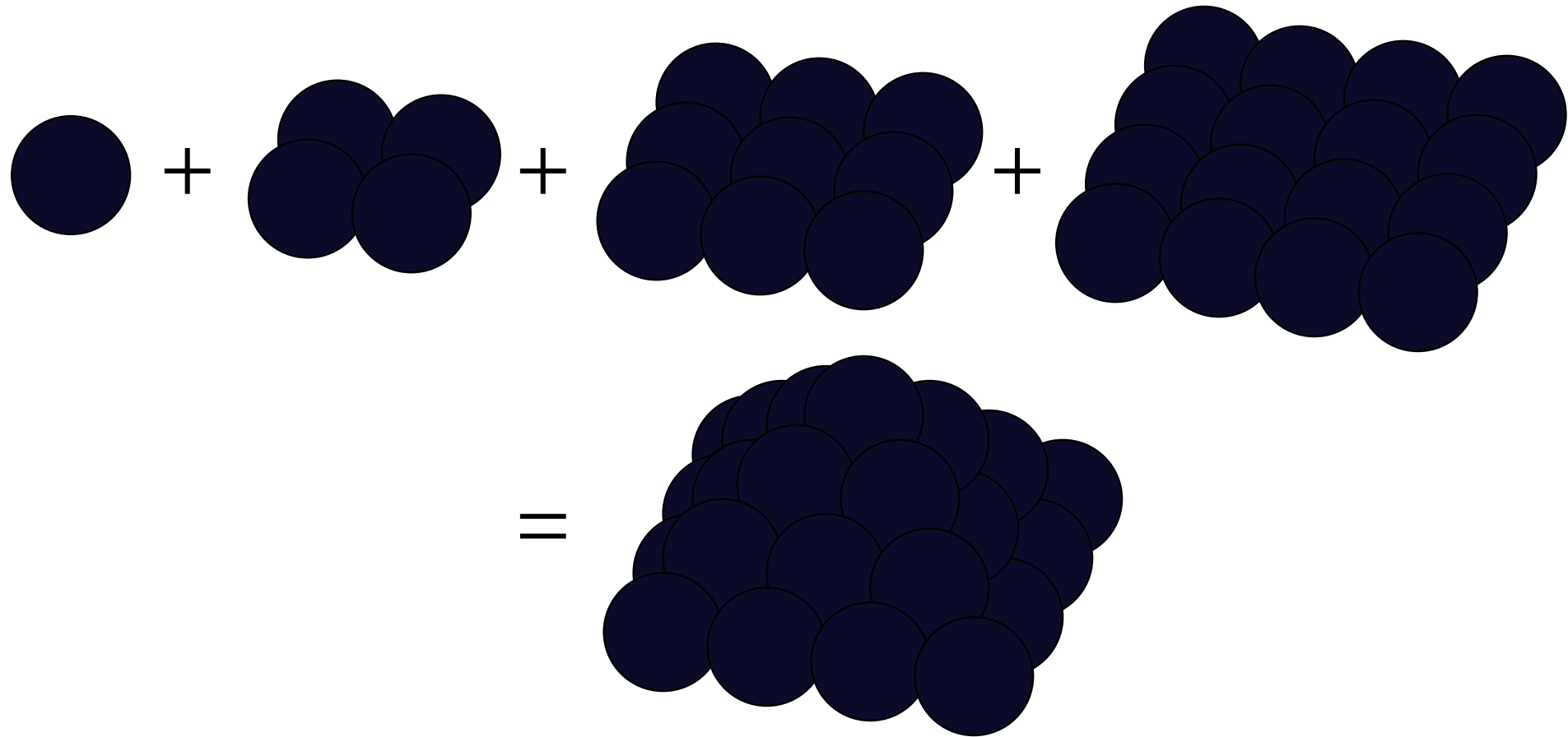


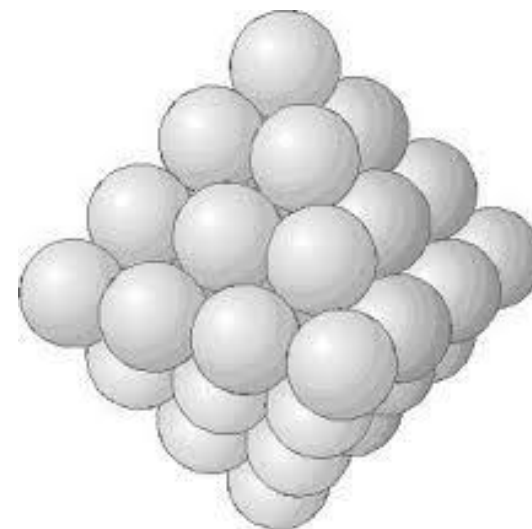
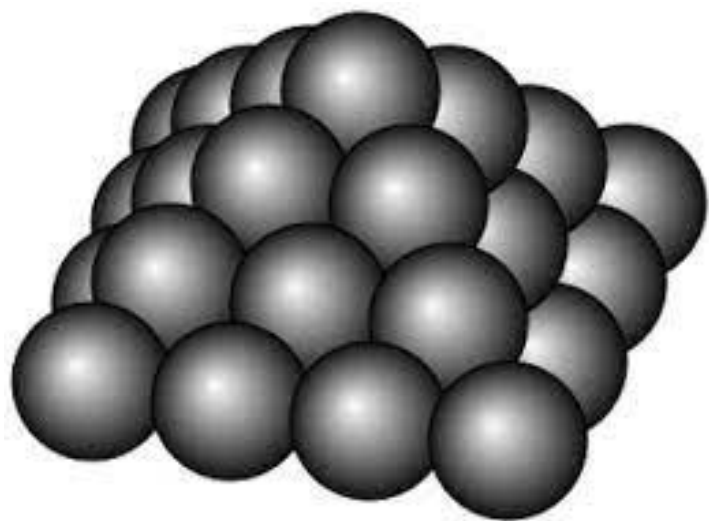
$$2 + 4 + 6 + \dots + 2024 = ?$$





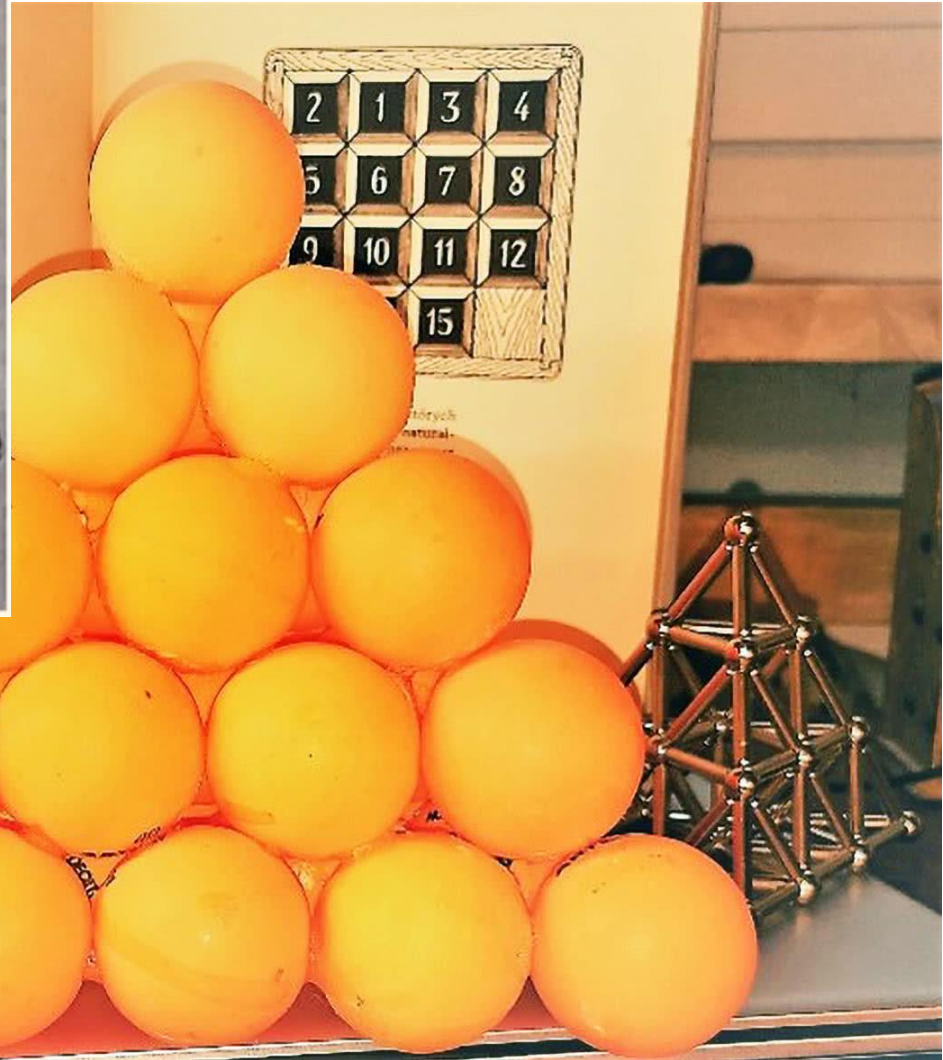
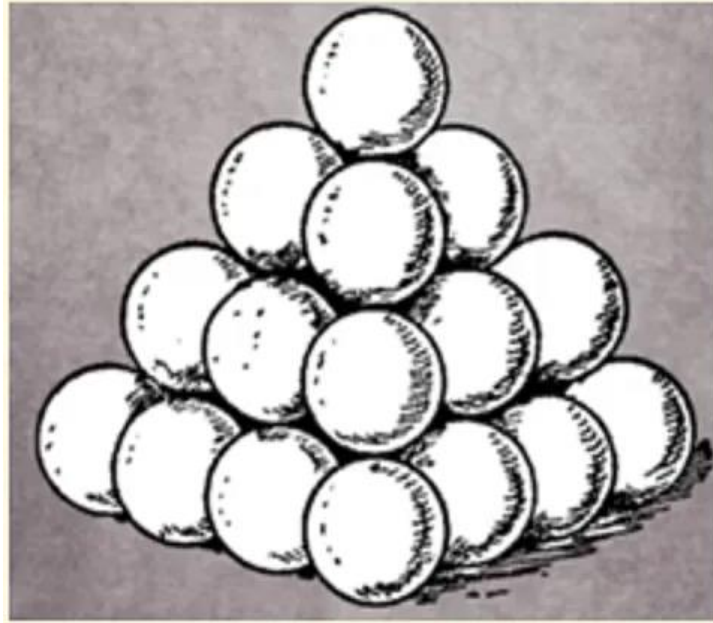
suma liczb kwadratowych = liczba piramidalna

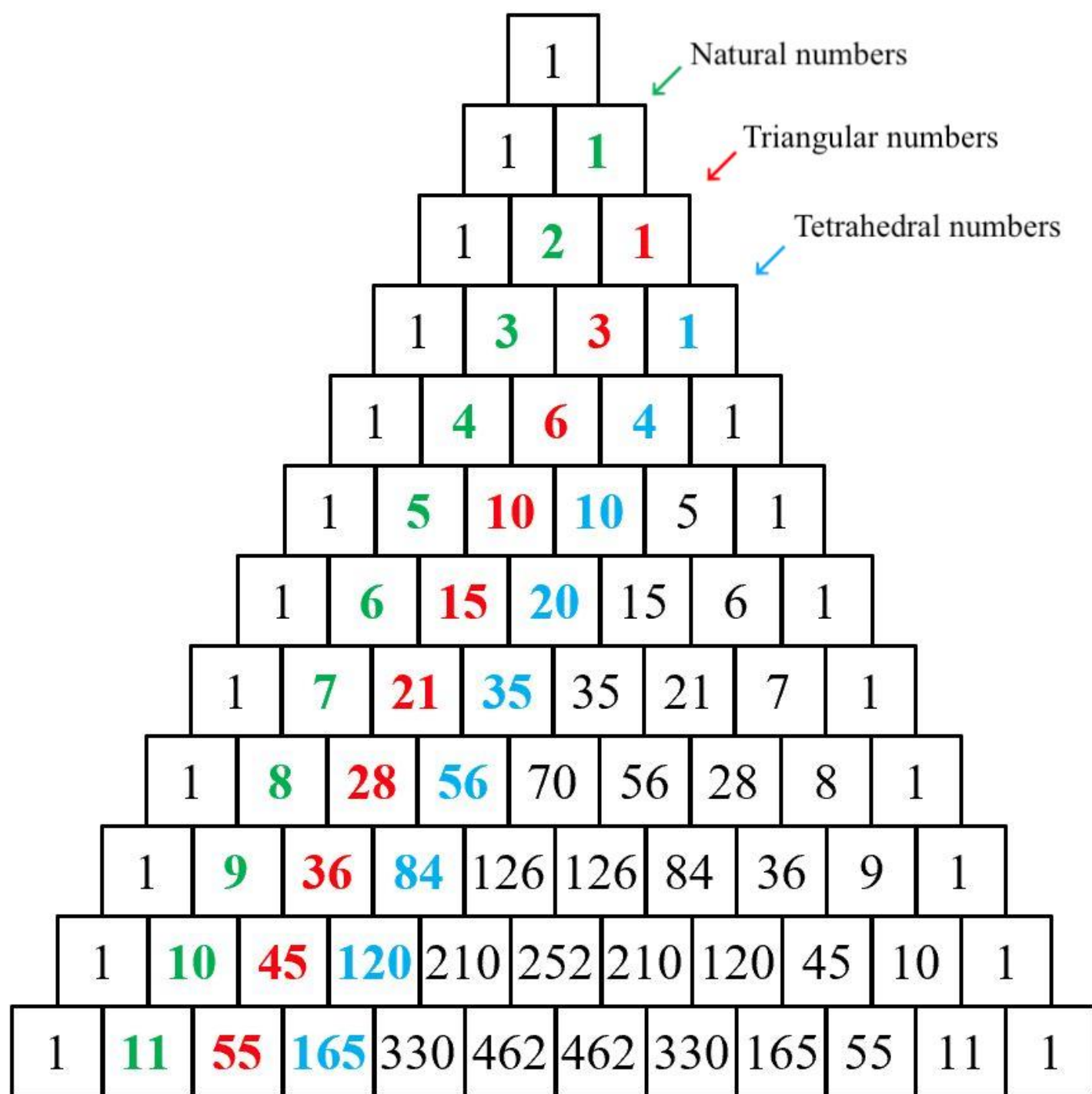




suma 2 kolejnych liczb piramidalnych =  
liczba ośmiościenna

suma liczb trójkątnych = liczba czworościenna

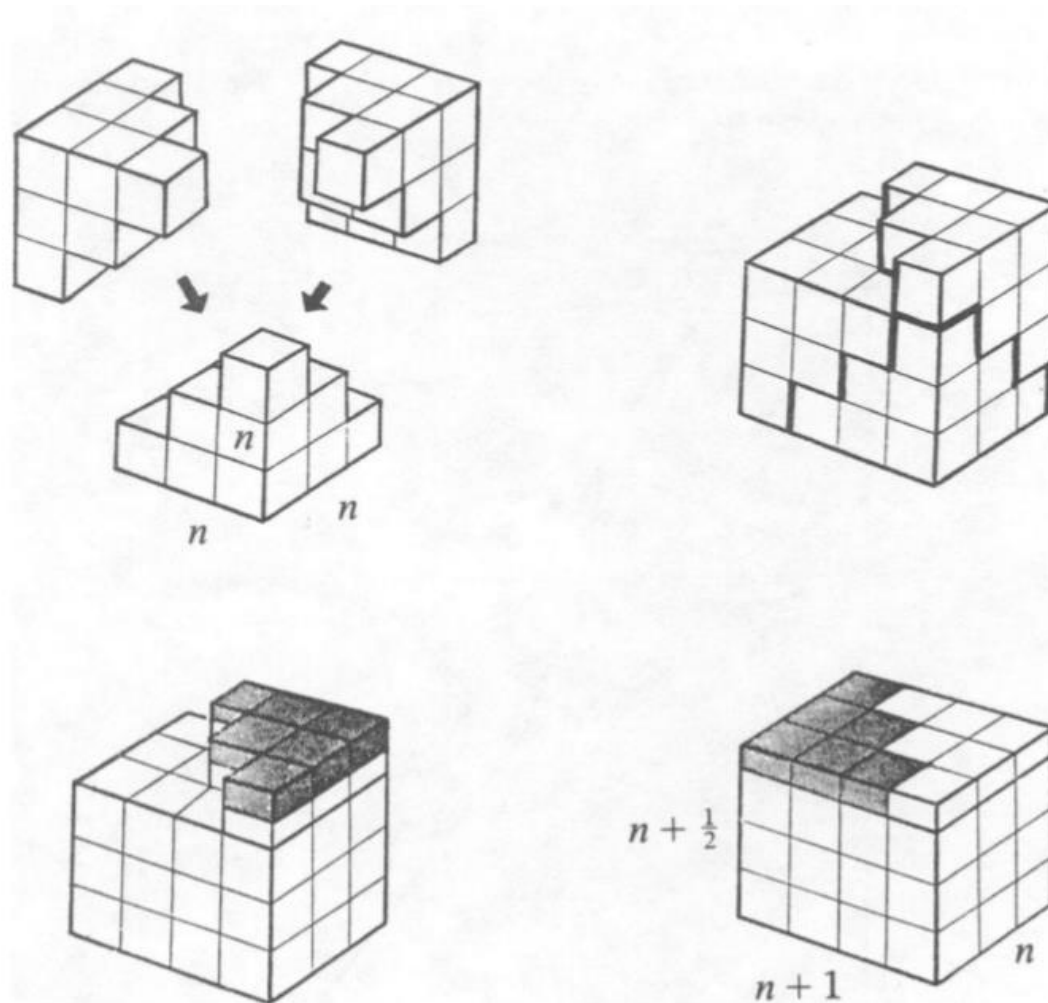




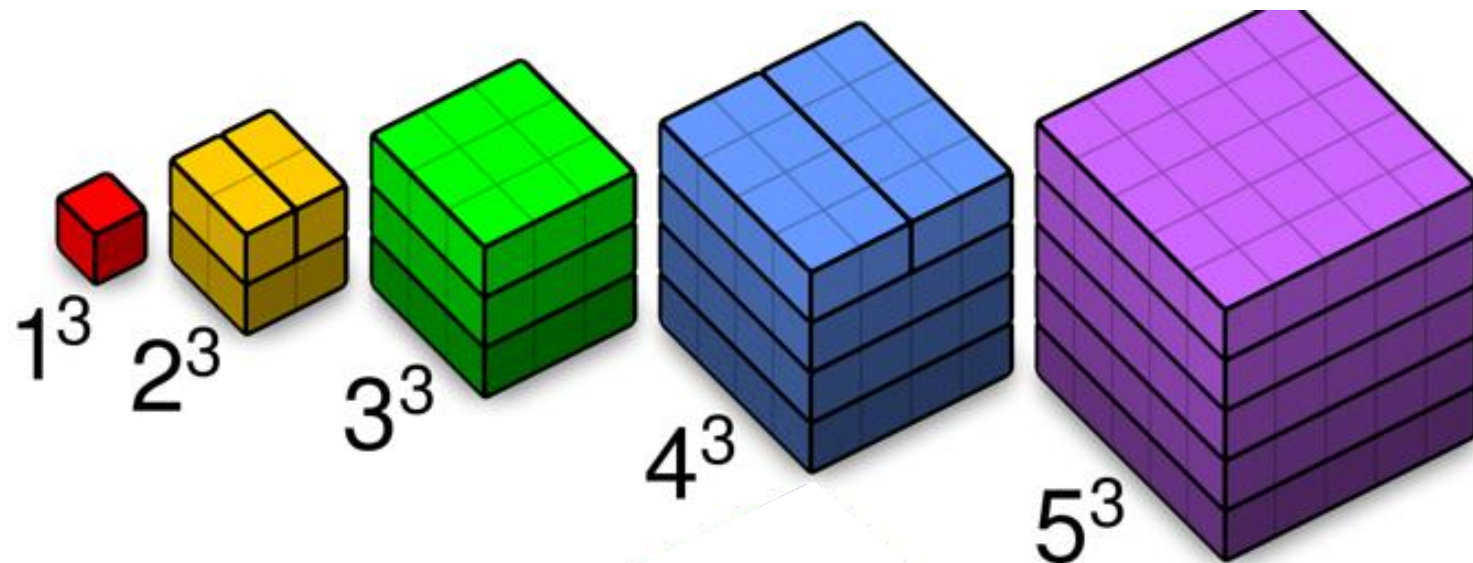
$$1^2 + 2^2 + 3^2 + \dots + 17^2 = ?$$

$$1^2 + 2^2 + \dots + n^2 = \frac{1}{3}n(n+1)(n+\frac{1}{2})$$

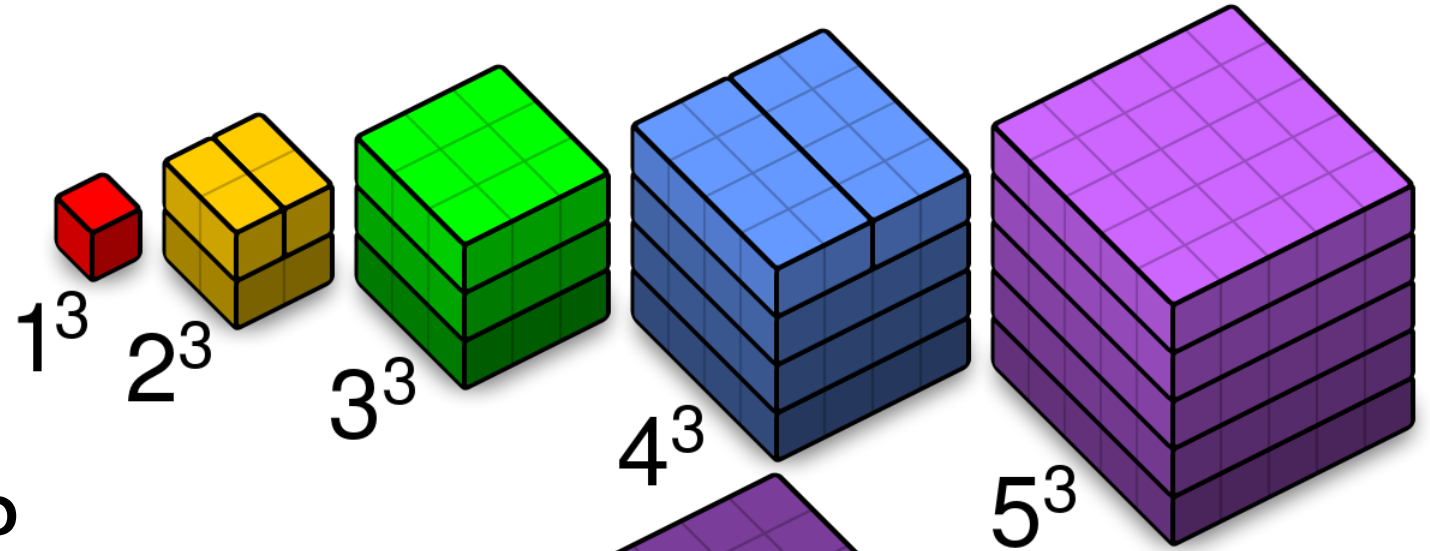
suma kwadratów  
kolejnych liczb  
naturalnych



# Liczby sześciennie

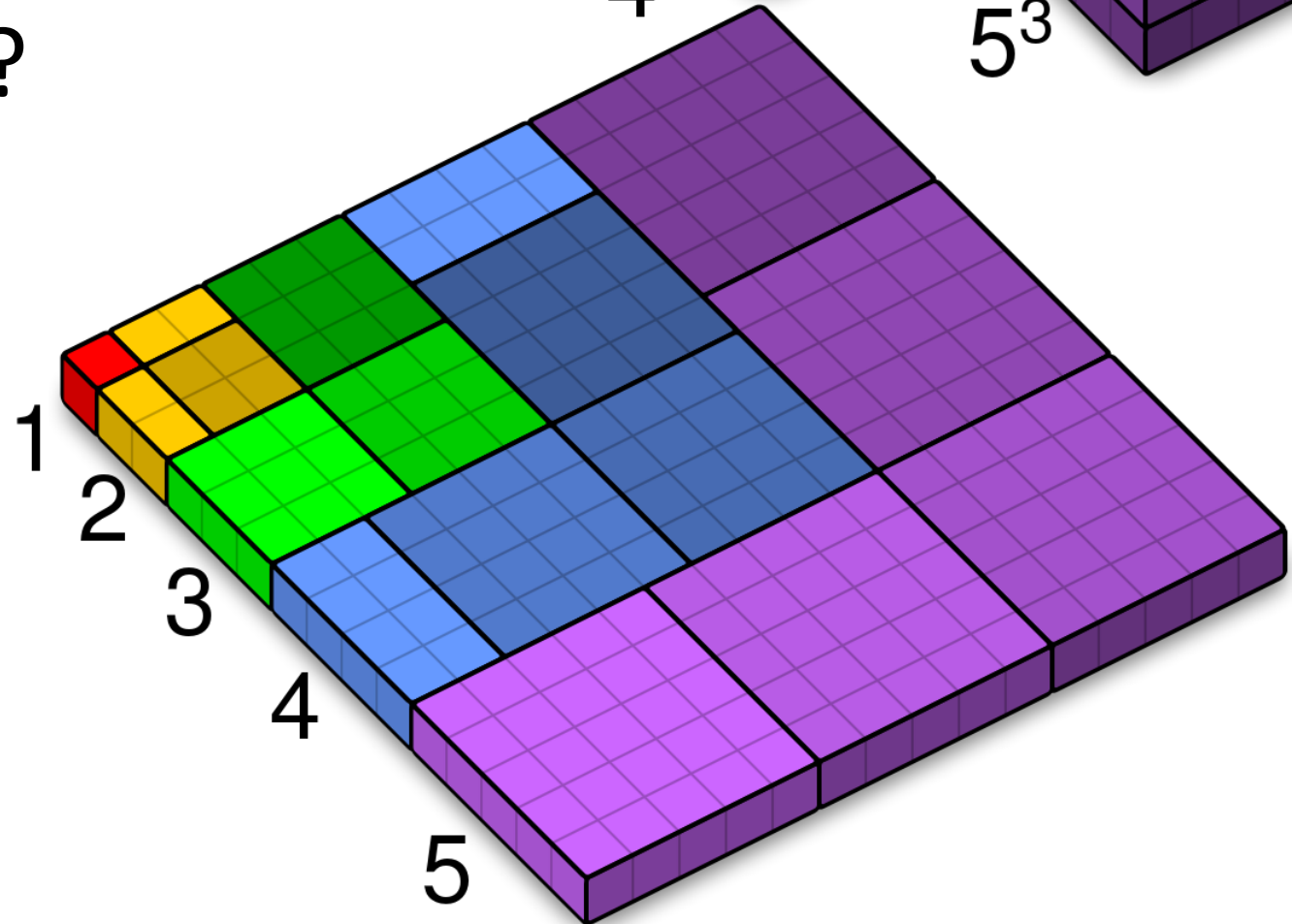






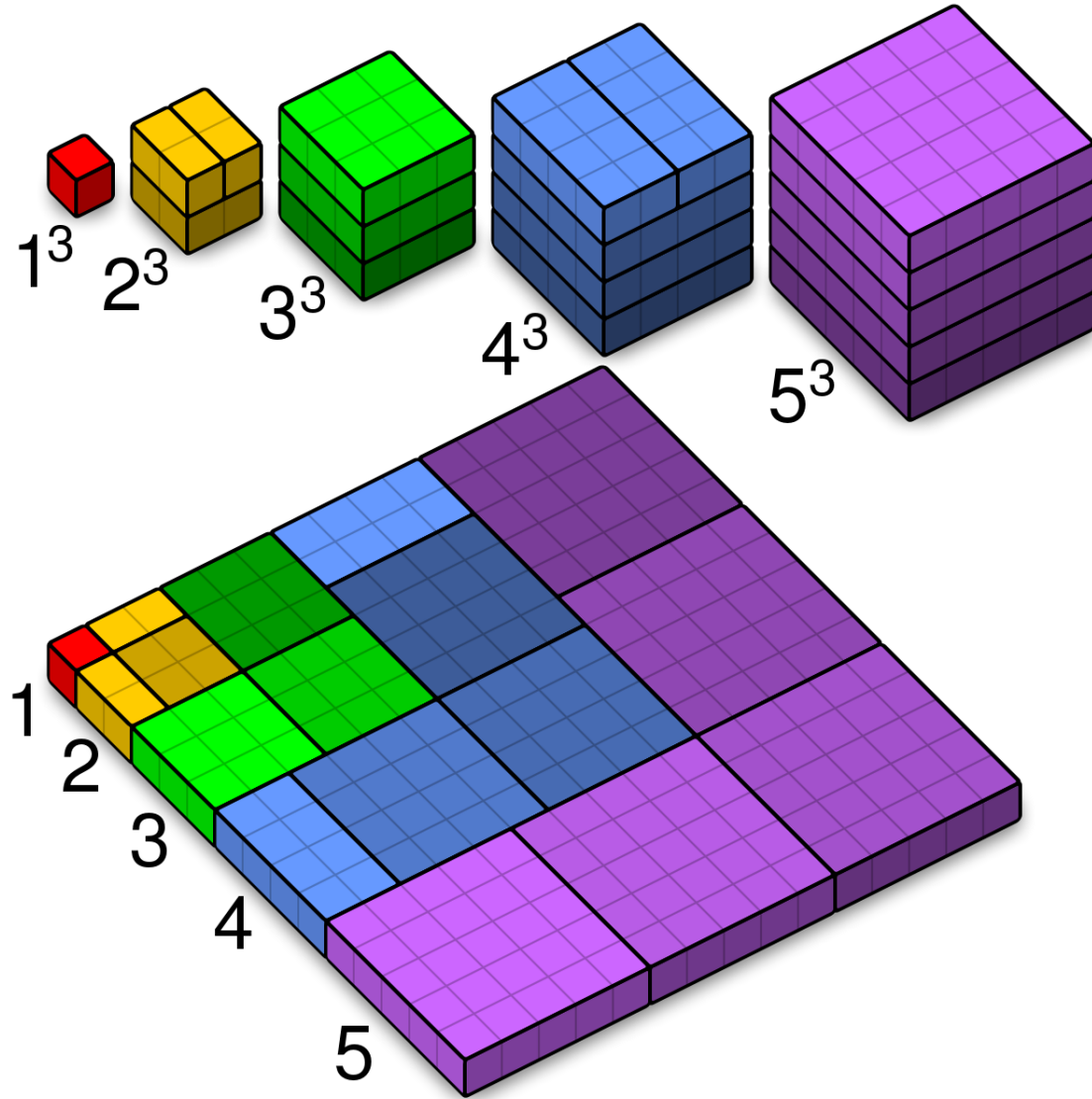
$$1^3 + 2^3 + 3^3 + \dots + 17^3 = ?$$

suma sześcianów  
kolejnych liczb  
naturalnych



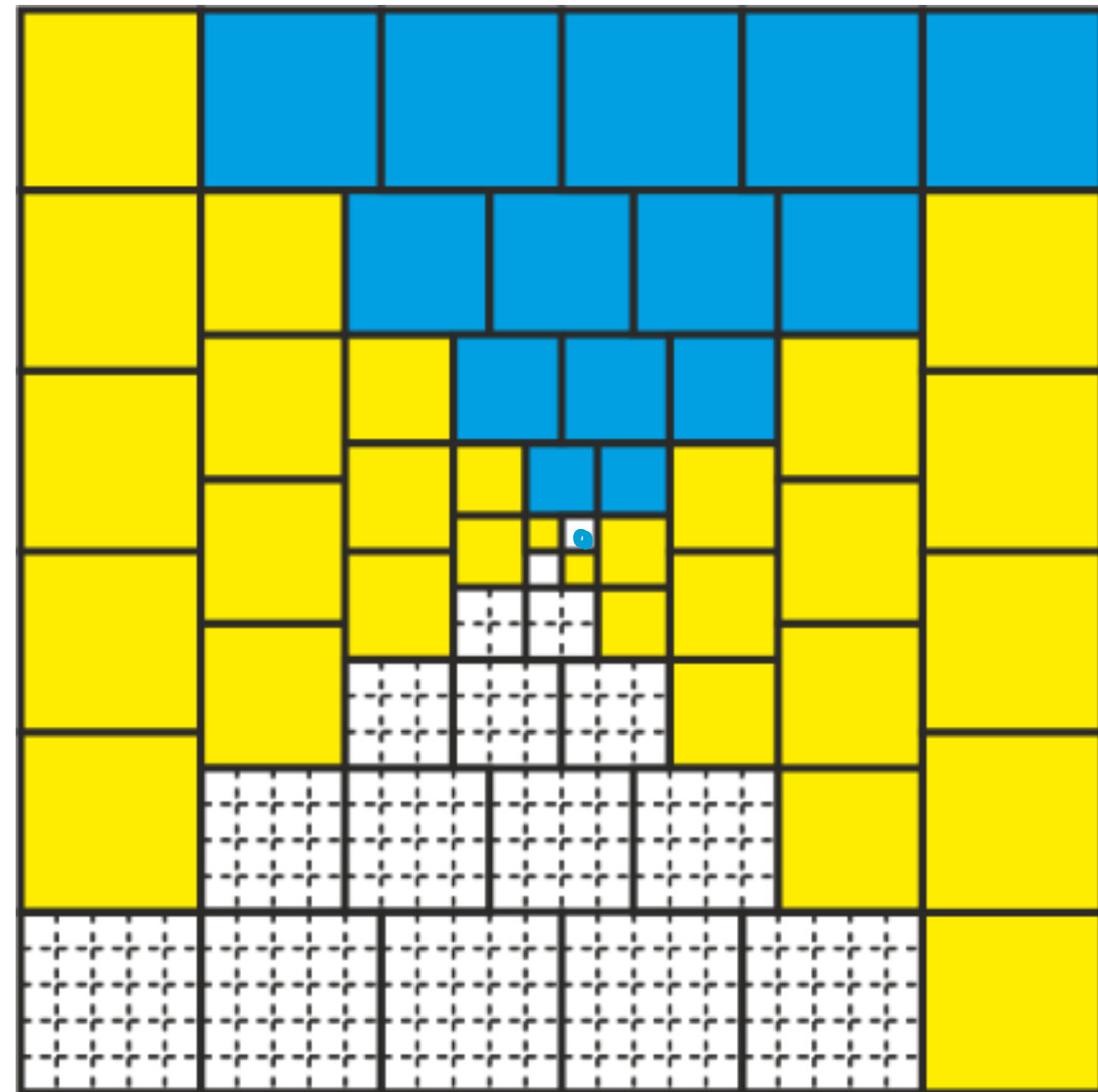


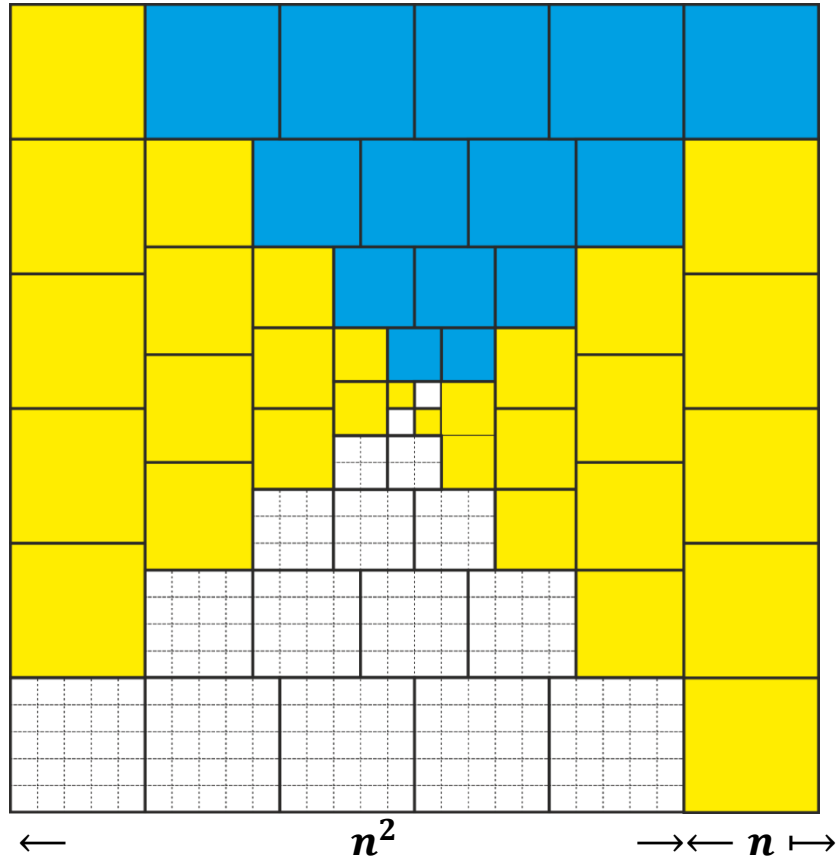
Suma sześciątów  
kolejnych liczb  
naturalnych



$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{1}{4} [n(n + 1)]^2$$

Suma sześciątów kolejnych liczb naturalnych





$$4 \cdot (1 \cdot 1^2 + 2 \cdot 2^2 + 3 \cdot 3^2 + \dots + 17 \cdot 17^2) = (n^2 + n)^2 = [n(n + 1)]^2$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{1}{4} [n(n + 1)]^2$$

Liczby  
pięciokątne

$$n = 1$$

$$P_1 = 1$$

$$n = 2$$

$$P_2 = 1 + 5 = 6$$

$$n = 3$$

$$P_3 = 1 + 5 + 10 = 16$$

$$n = 4$$

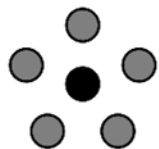
$$P_4 = 1 + 5 + 10 + 15 = 31$$

$$n = 5$$

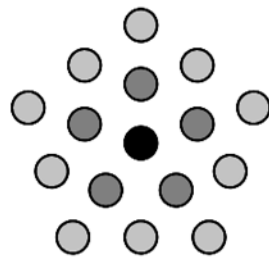
$$P_5 = 1 + 5 + 10 + 15 + 20 = 51$$



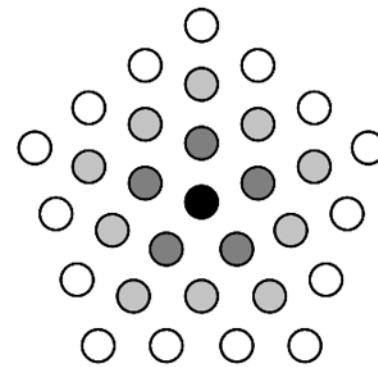
1



6



16



31

Liczby  
sześciokątne

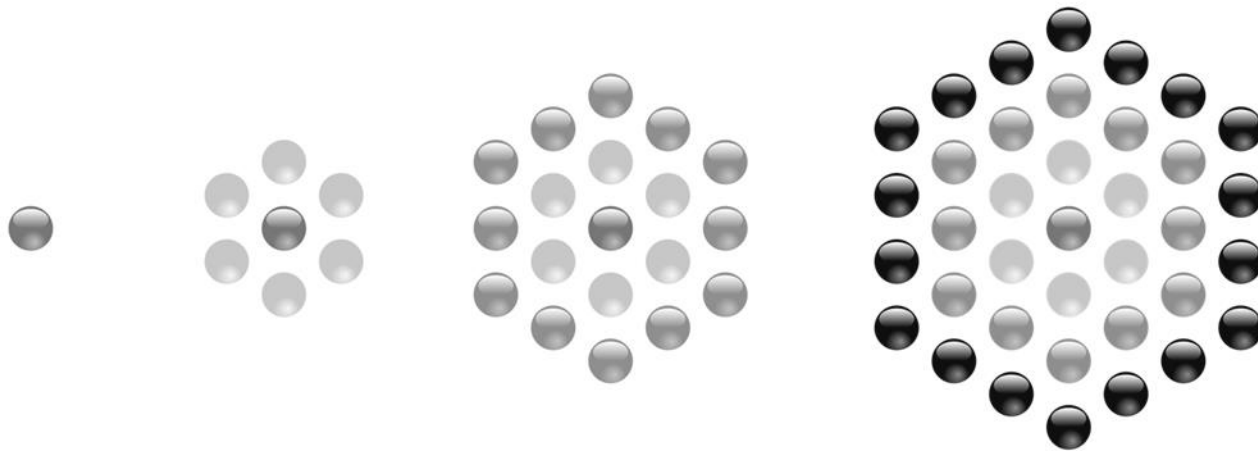
$$n = 1 \quad H_1 = 1$$

$$n = 2 \quad H_2 = 1 + 6 = 7$$

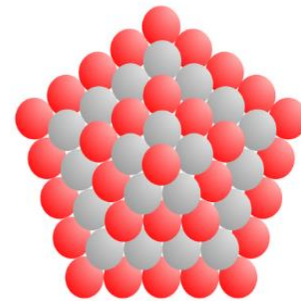
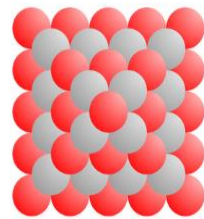
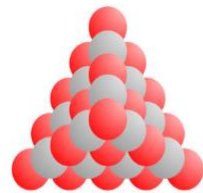
$$n = 3 \quad H_3 = 1 + 6 + 12 = 19$$

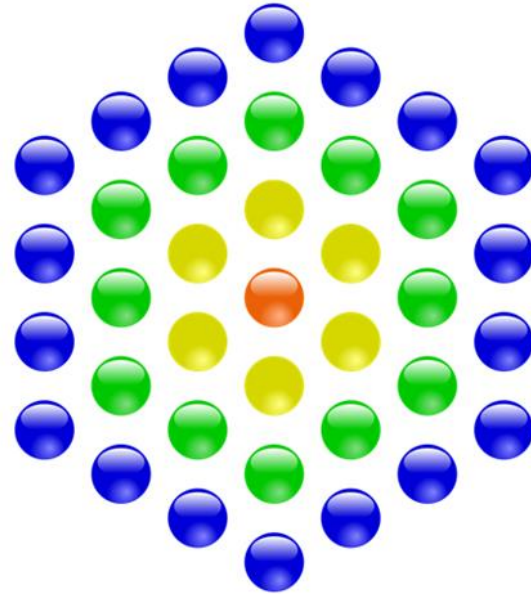
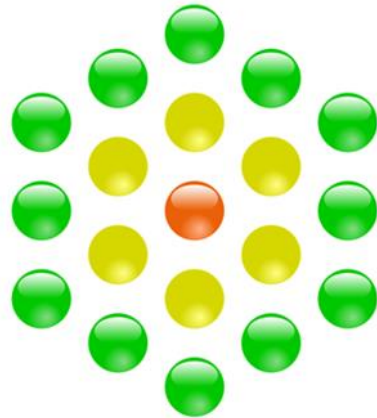
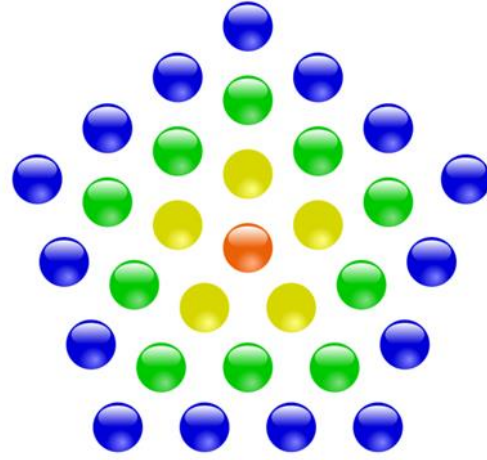
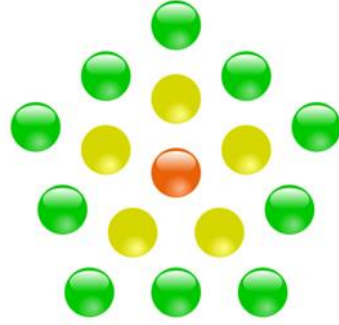
$$n = 4 \quad H_4 = 1 + 6 + 12 + 18 = 37$$

$$n = 5 \quad H_5 = 1 + 6 + 12 + 18 + 24 = 61$$



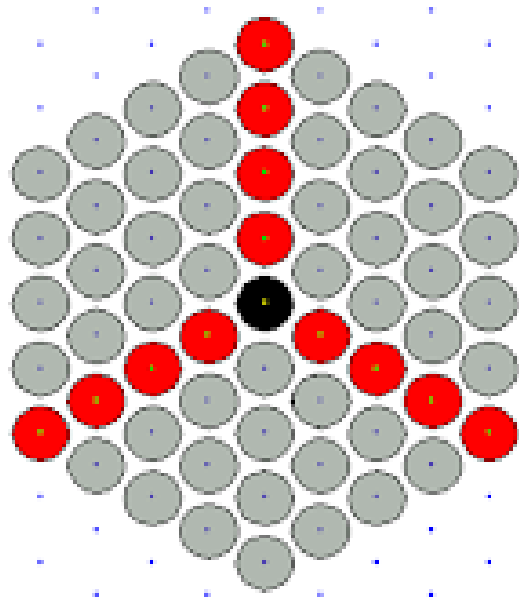
# Liczby piramidalne (trójkątne, kwadratowe, pięciokątne)







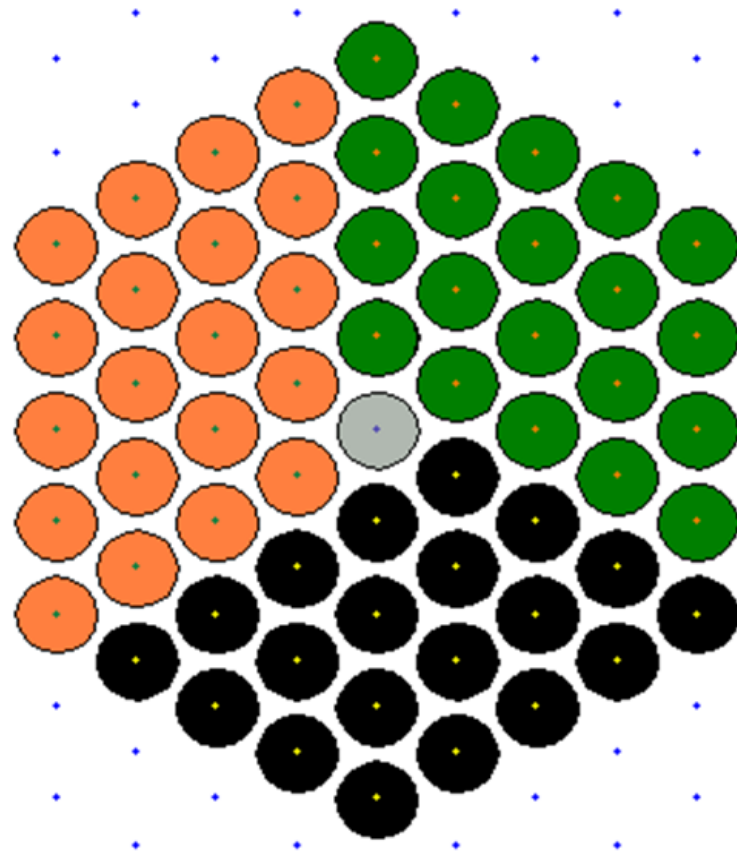
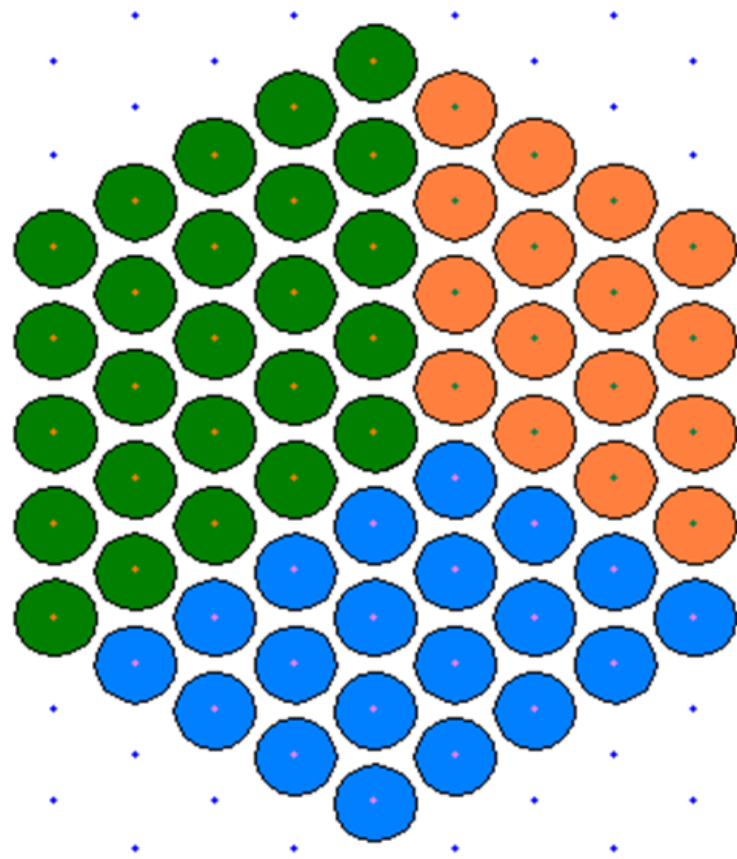
liczby sześciokątne



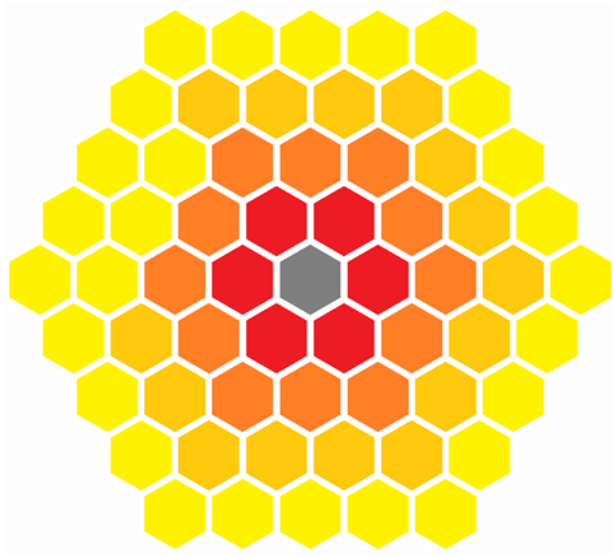
$n = 5$

$$3n^2 - 3n + 1$$

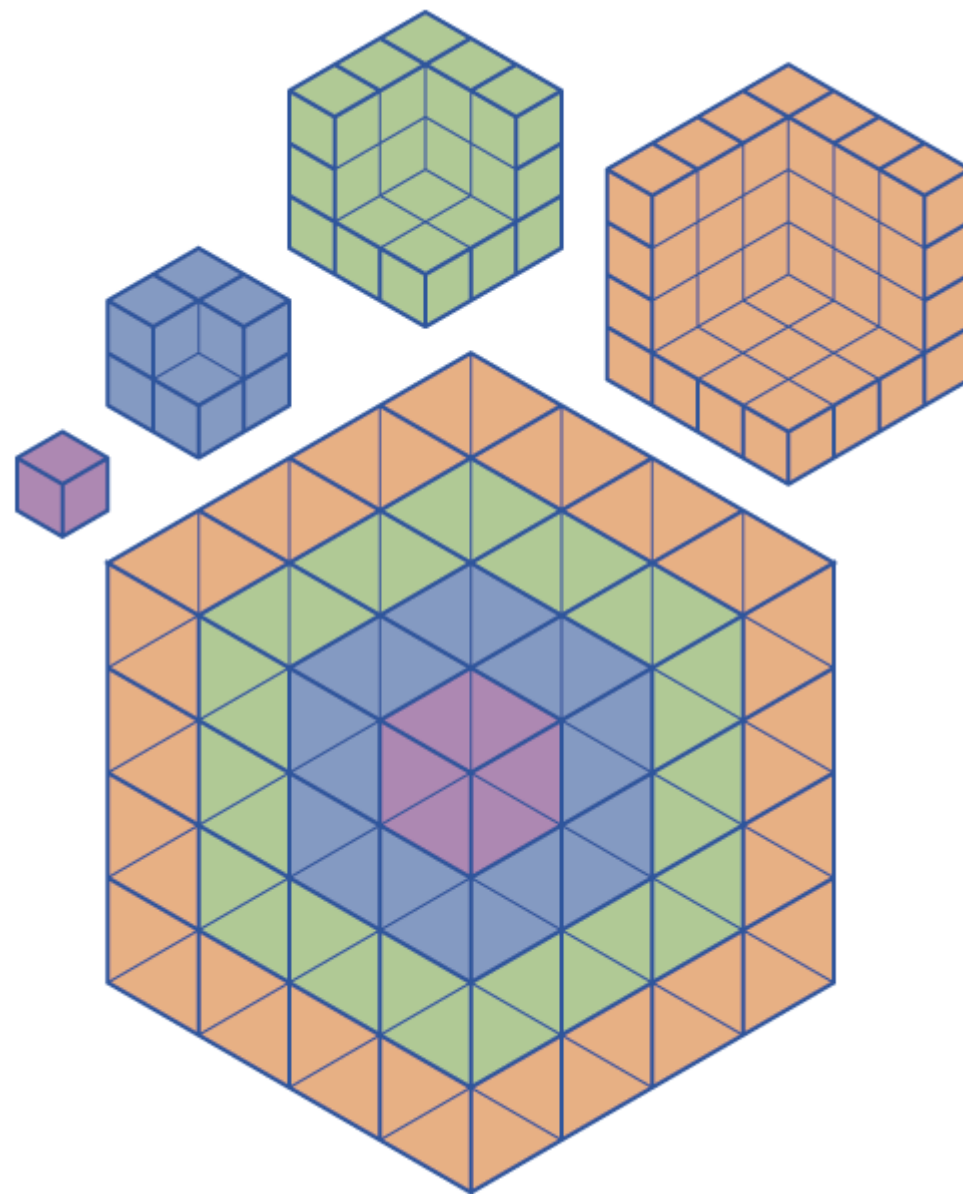
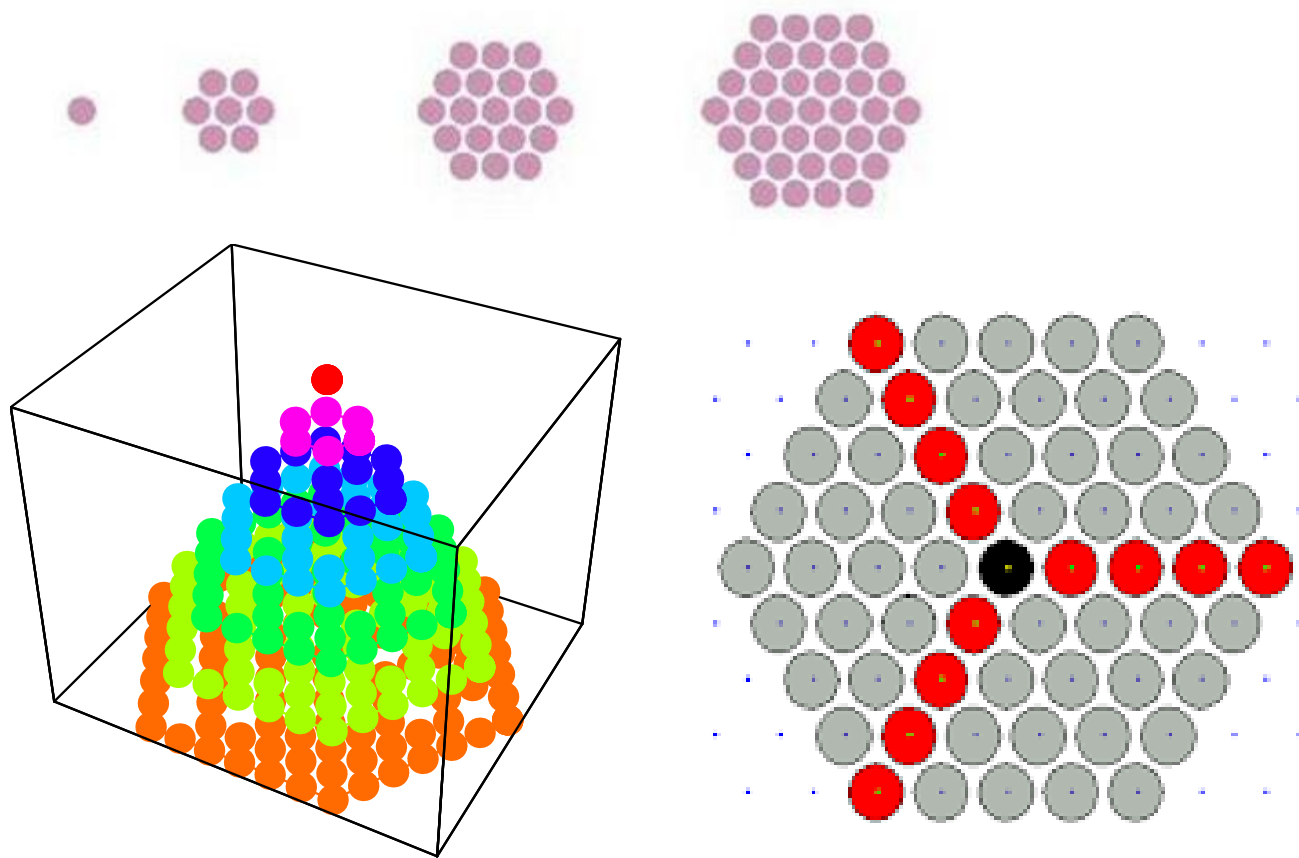
$n = 5$

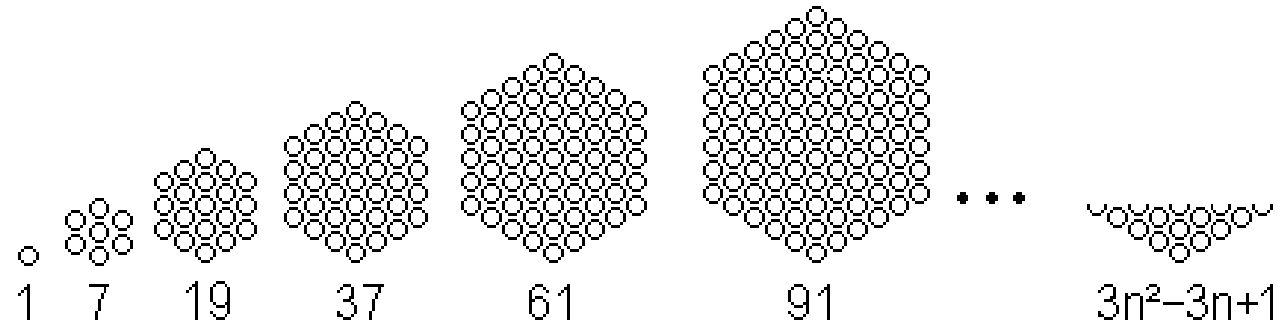






liczba piramidalna sześciokątna =  
suma liczb sześciokątnych

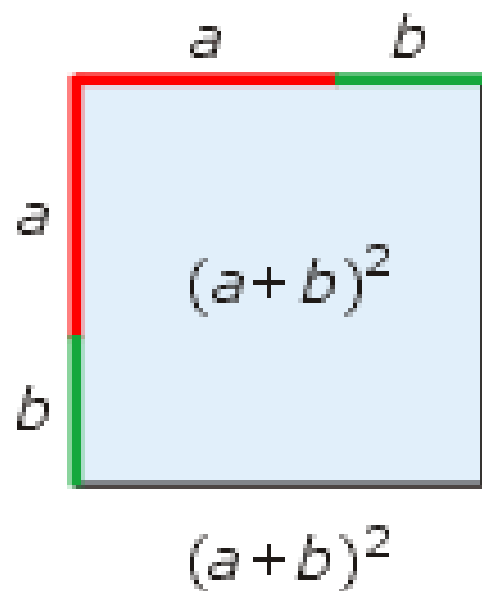




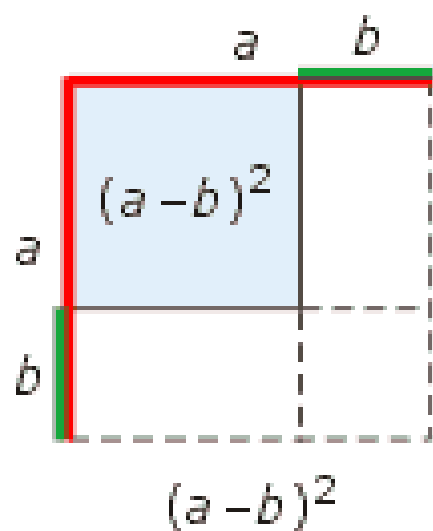
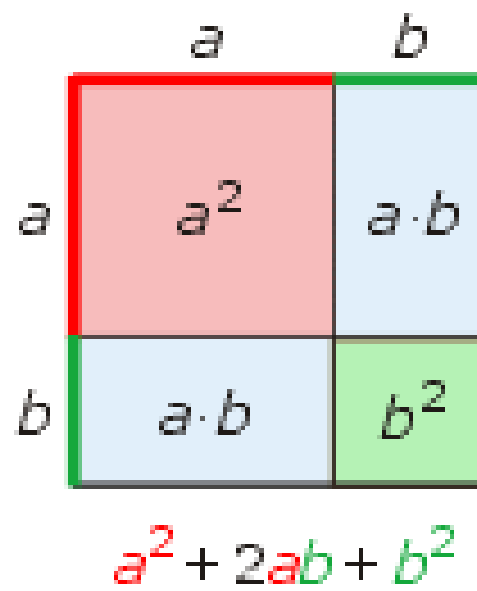
$$n^3 - (n-1)^3 = n^3 - n^3 + 3n^2 - 3n + 1 = 3n^2 - 3n + 1$$



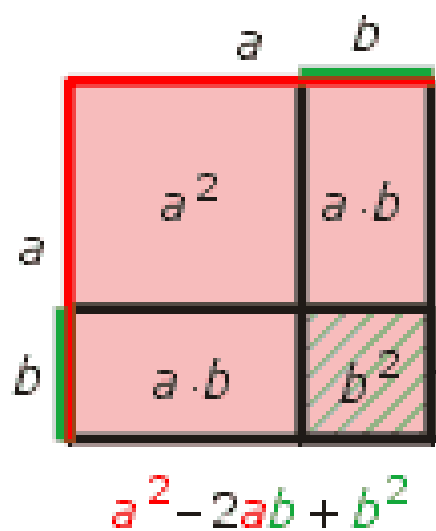
# Rysunkowe wzory z algebry



=

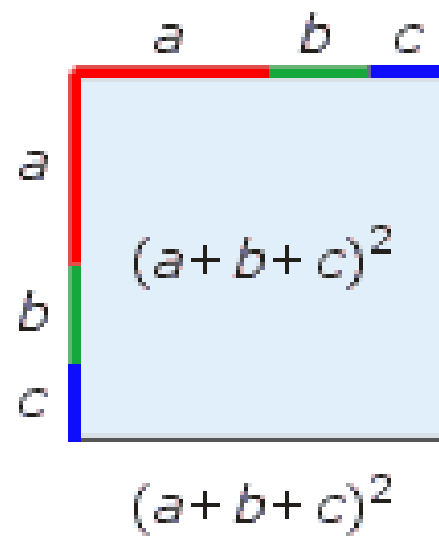


=

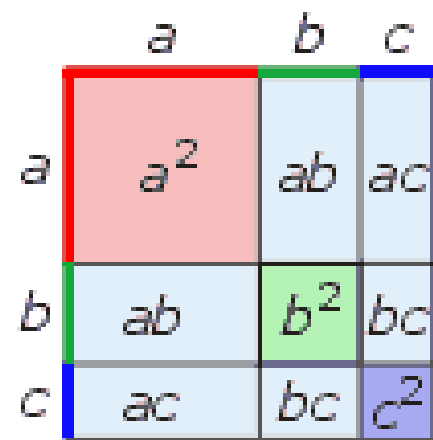


ten kawałek  
był wyrzucony 2 razy,  
więc trzeba go dodać

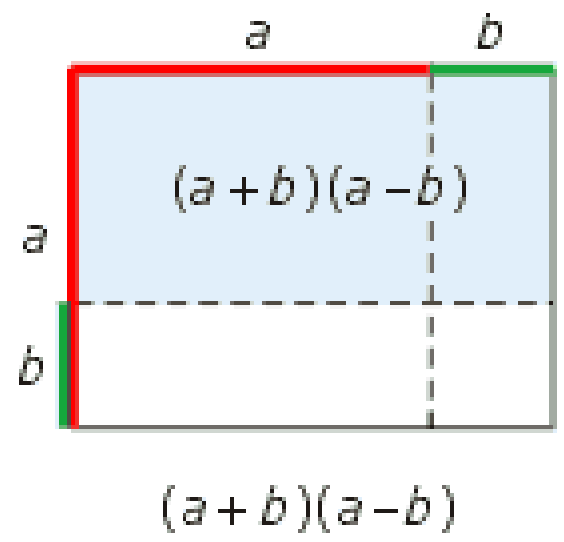




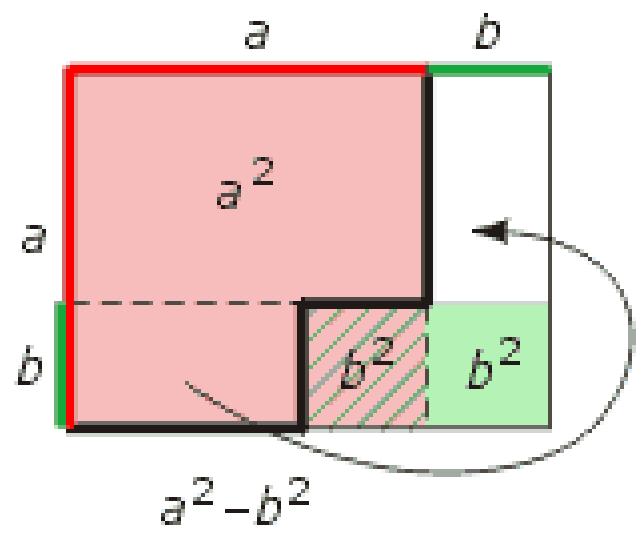
=



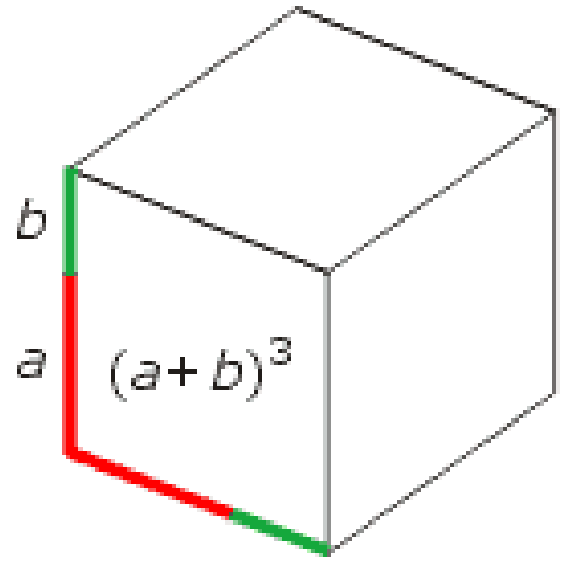
$= a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$



=

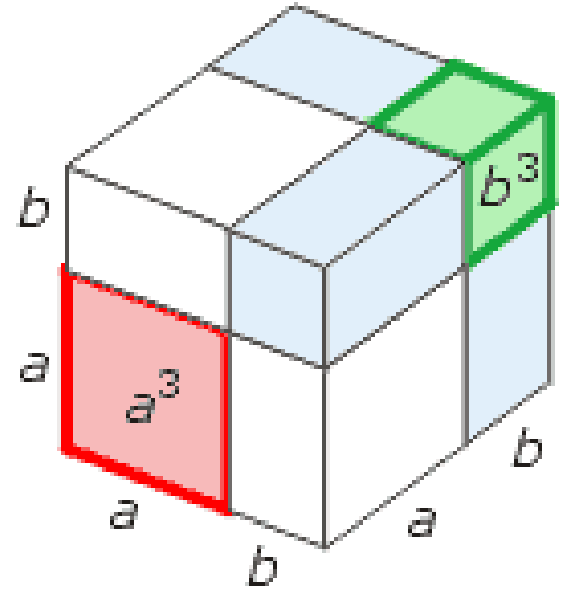


=



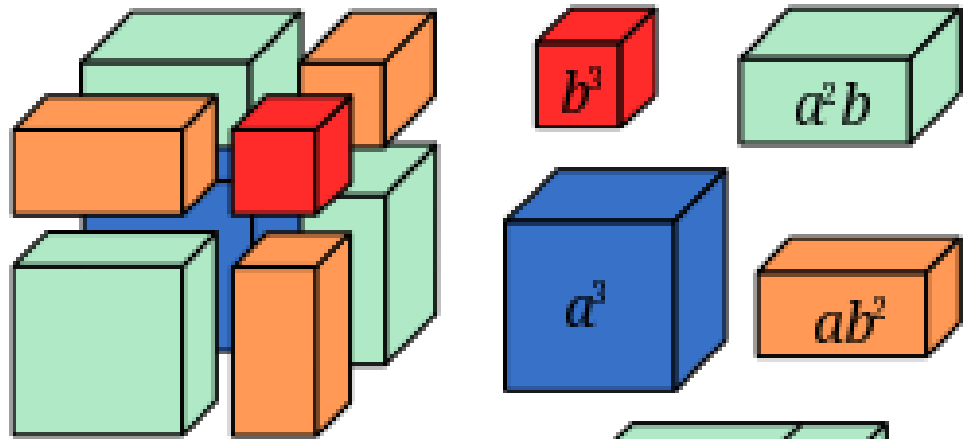
$$(a+b)^3$$

=

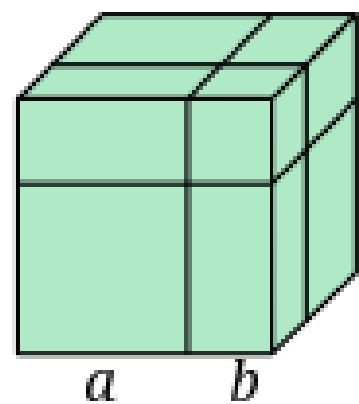


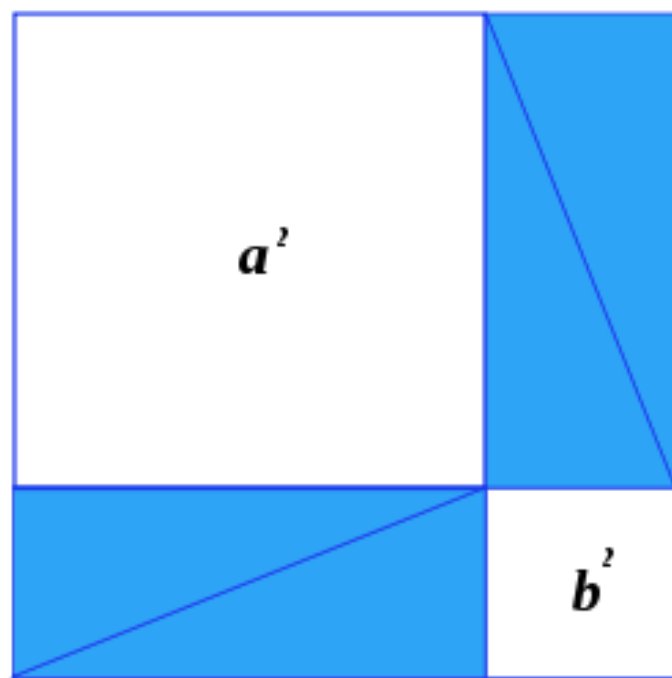
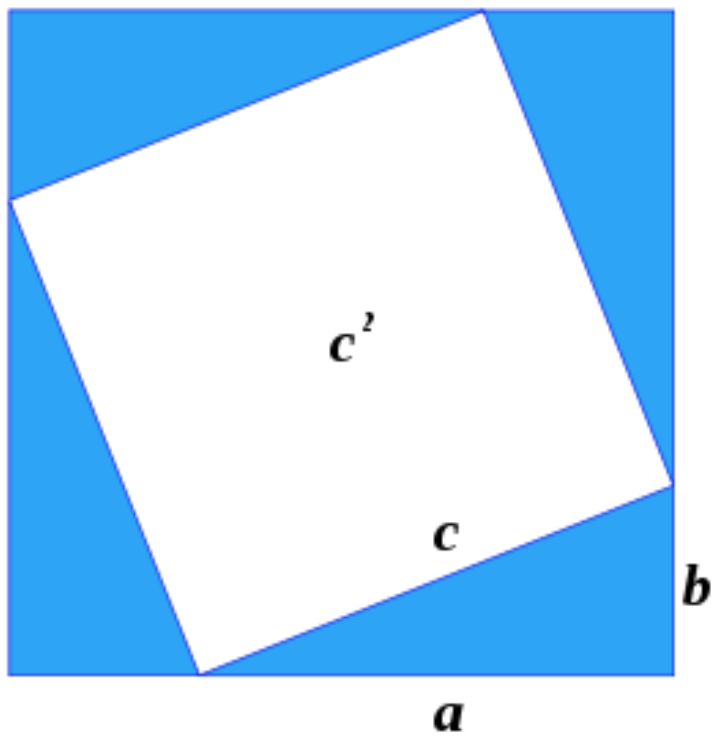
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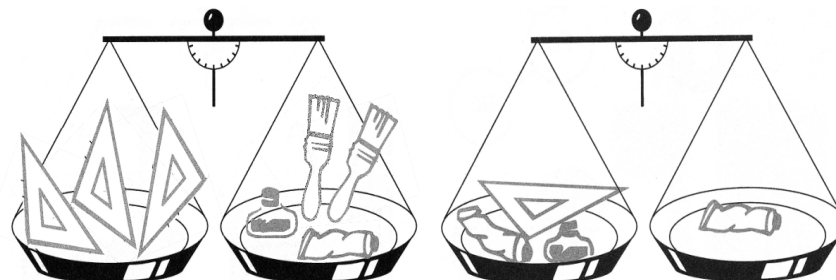
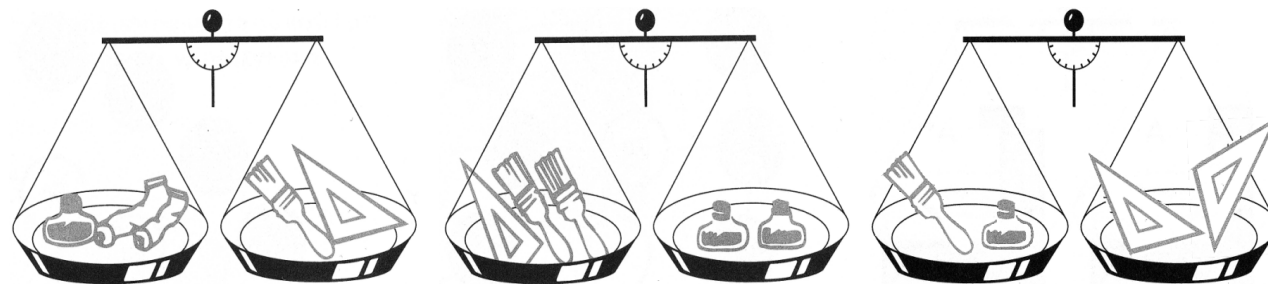
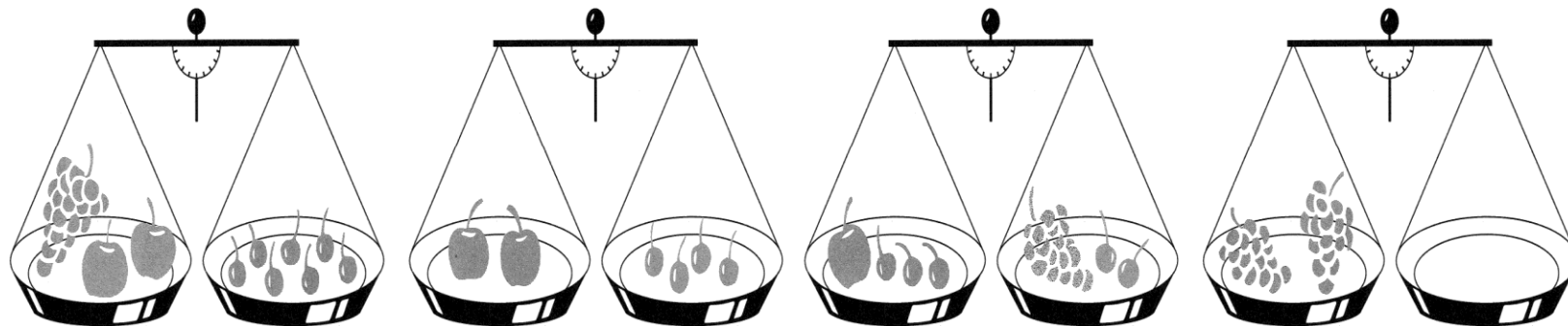
$$a^3 + b^3 + 3a^2b + 3b^2a$$



$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$







# ALGEBRAFY

Algebrą to łamigłówka arytmetyczna, w której cyfry zastąpiono literami. W algebrze obowiązuje zasada, w której takim samym literom odpowiada taka sama cyfra, a różnym literom odpowiadają różne cyfry. Nieco inny charakter mają zadania, w których zamiast cyfr występują symbole  $\square$  albo  $*$ .

1. Rozszyfruj, jakie cyfry kryją się pod literami  $A$ ,  $B$ ,  $C$  w każdym z równań:

a)  $AA + B = BCC$ ,    b)  $AA + AB = CCC$ ,    c)  $AC + BC = CCA$ .

$AA \quad B \quad AB \quad A \quad BC \quad CC \quad C$

2. Trzycyfrowe liczby  $AAA$ ,  $AAB$ ,  $ACC$  spełniają warunek

$$AAA + AAB + ACC = 2003.$$

Jakie cyfry kryją się po literami  $A$ ,  $B$ ,  $C$ ?

3. Jakie cyfry kryją się pod literami  $A$ ,  $B$ ,  $C$ , jeśli

$$\begin{array}{r} ABC \\ +ACB \\ \hline 1765 \end{array}$$

1765

4. Przeanalizuj każdą równość osobno i odpowiedz na pytanie: Jakie cyfry są ukryte pod literami  $A, B, C$ ?

a)  $A \cdot AB = CCC$ ,    b)  $A + A + BB = CCC$ ,    c)  $ABC \cdot AB = 2002$ .

5. W wyniku dodania liczby  $ABC$  do liczby  $ABC$  otrzymano liczbę  $DAD$ . Jakie cyfry znajdują się pod literami  $A$ ,  $B$ ,  $C$ ,  $D$ ?

6. Odtwórz dodawanie i mnożenie:

a) 
$$\begin{array}{r} * * \\ + * * \\ \hline 197 \end{array},$$

b) 
$$\begin{array}{r} 6 * \\ \cdot * * \\ \hline * * 6 \end{array},$$

c) 
$$\begin{array}{r} * * * \\ \cdot * 5 \\ \hline 2005 \\ * * * \\ \hline * * * * * \end{array}.$$

USTA  
+ USTA

---

CAŁUS

TRAF  
TRAF  
TRAF  
+ TRAF

---

FART

TRZYZERO:ZERO=ZERO



Uniwersytet  
Wrocławski

MATMA  
+ STUDIA  

---

SUKCES

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