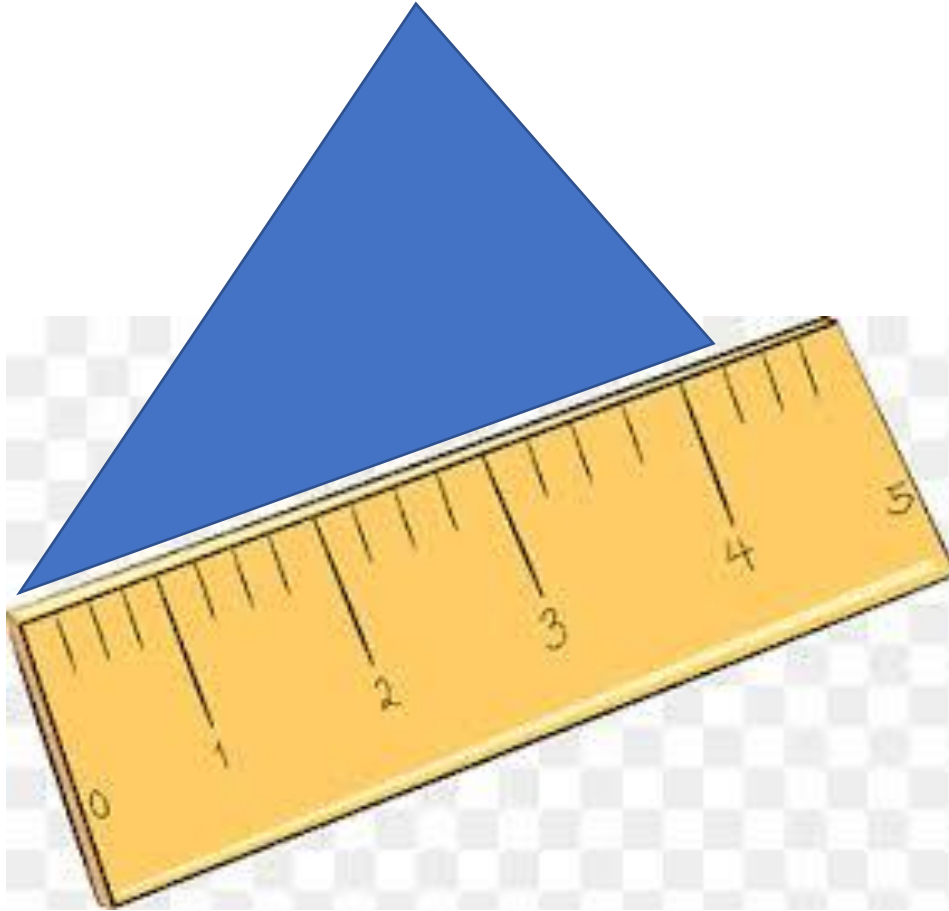
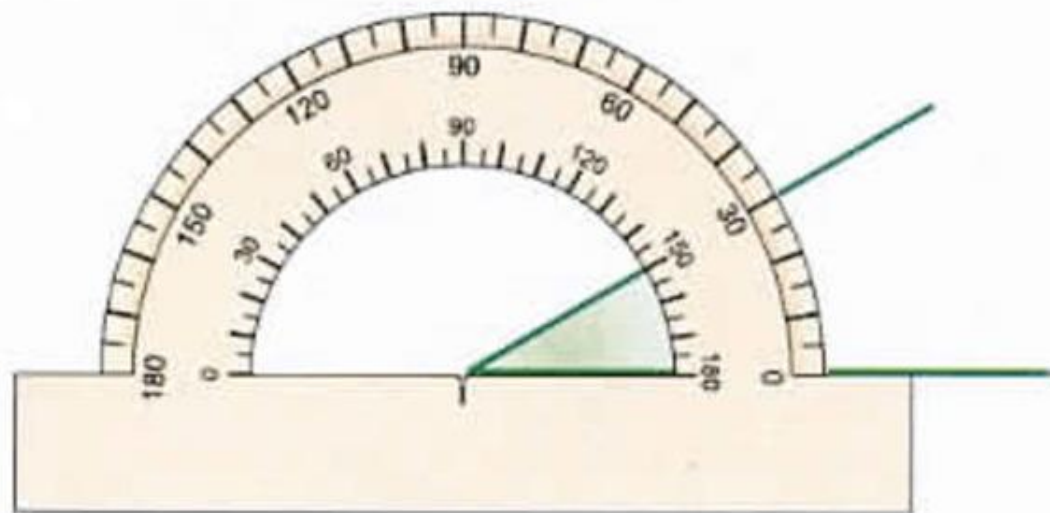


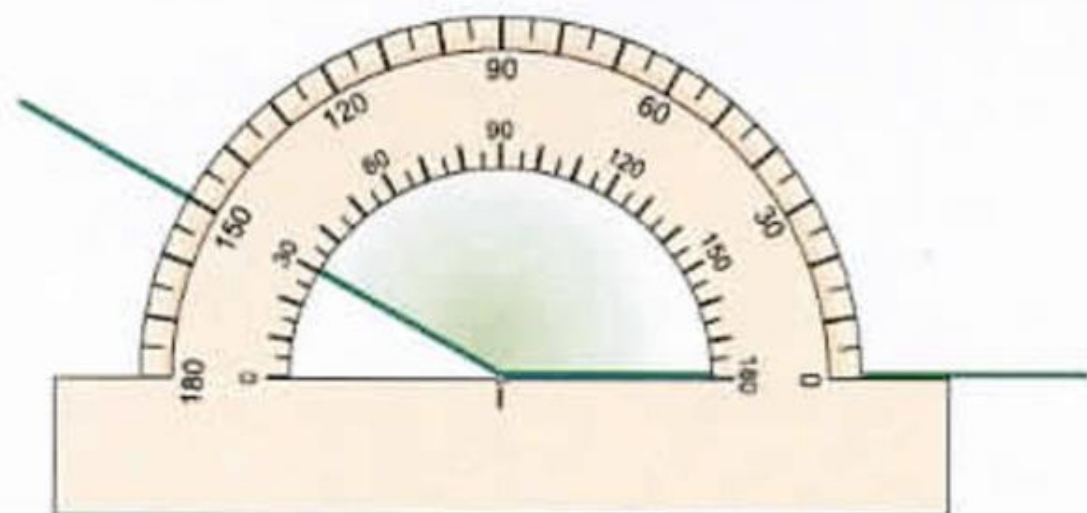
CZYM JEST, A CZYM NIE JEST  
TRYGONOMETRIA

TRI GON METRON

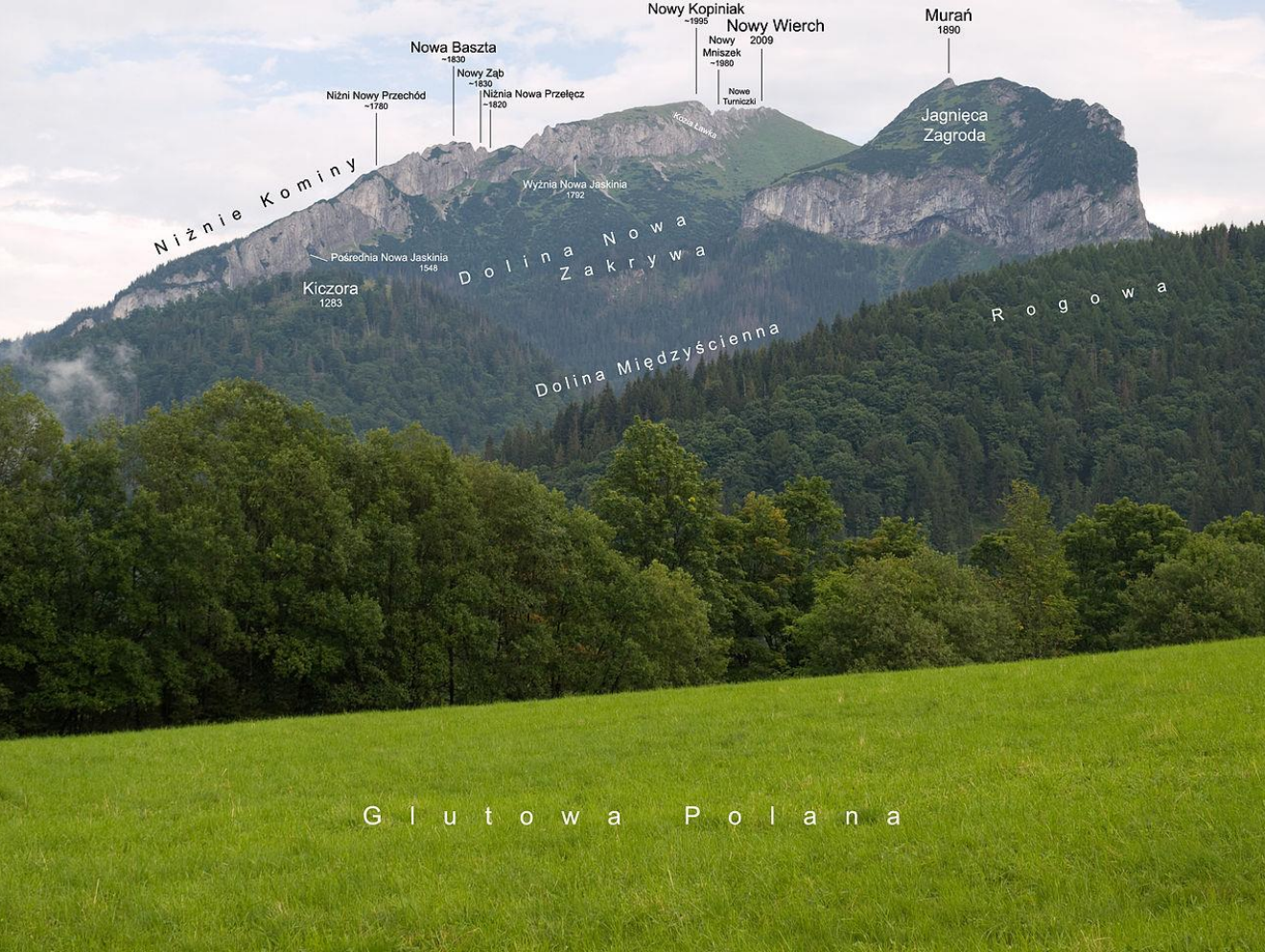




..... $30^{\circ}$ .....



..... $150^{\circ}$ .....



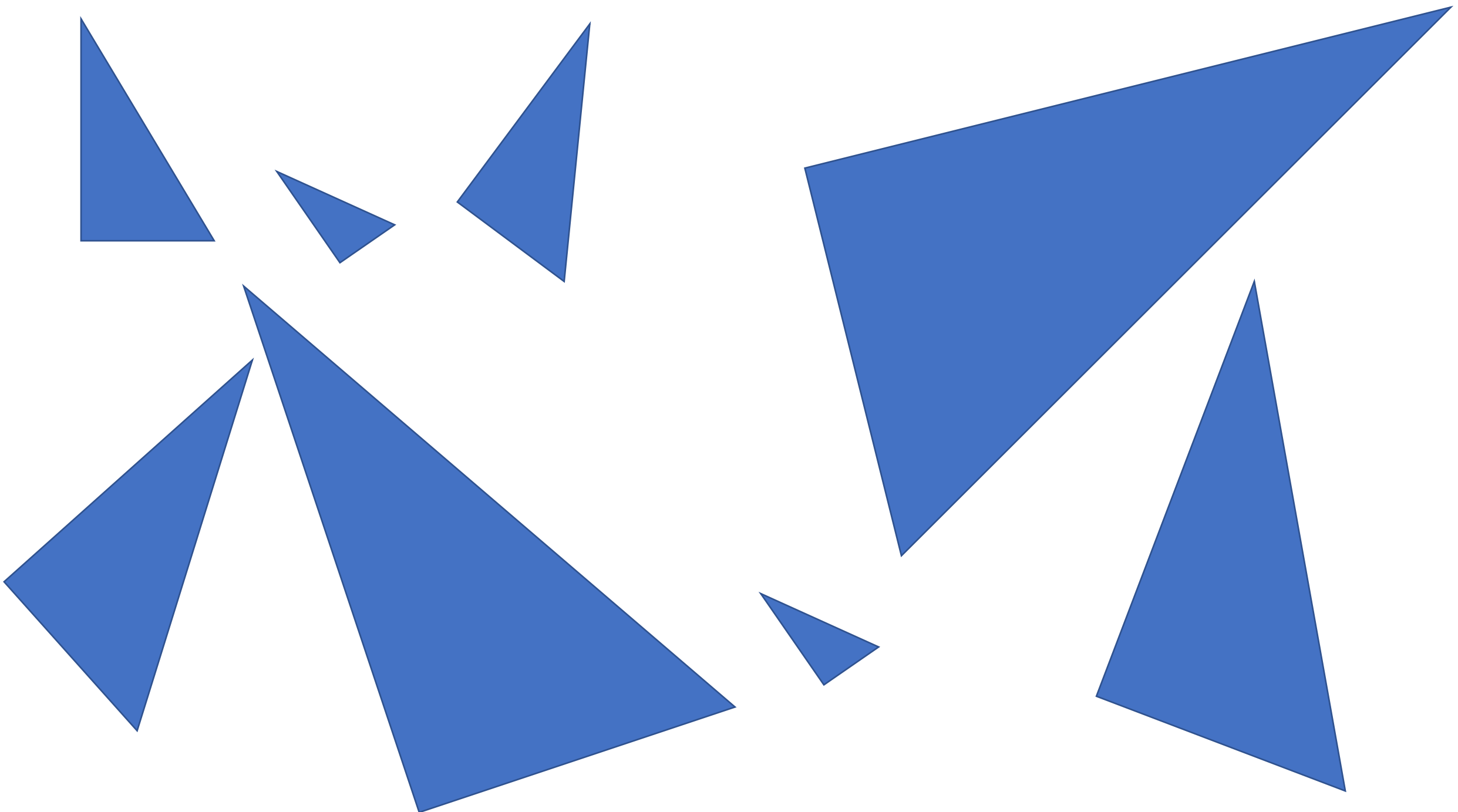


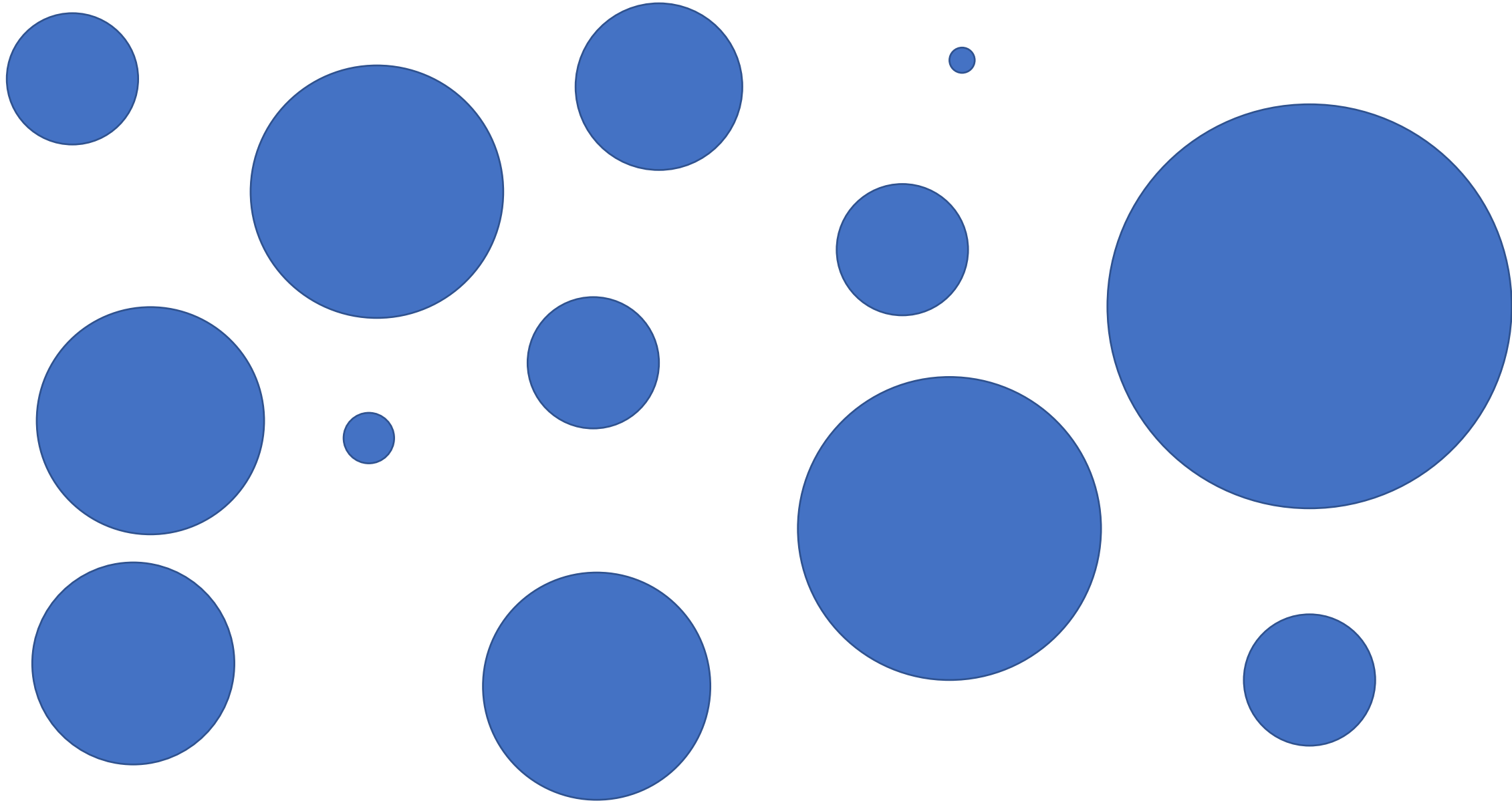


**teodolit  
geodezyjny**

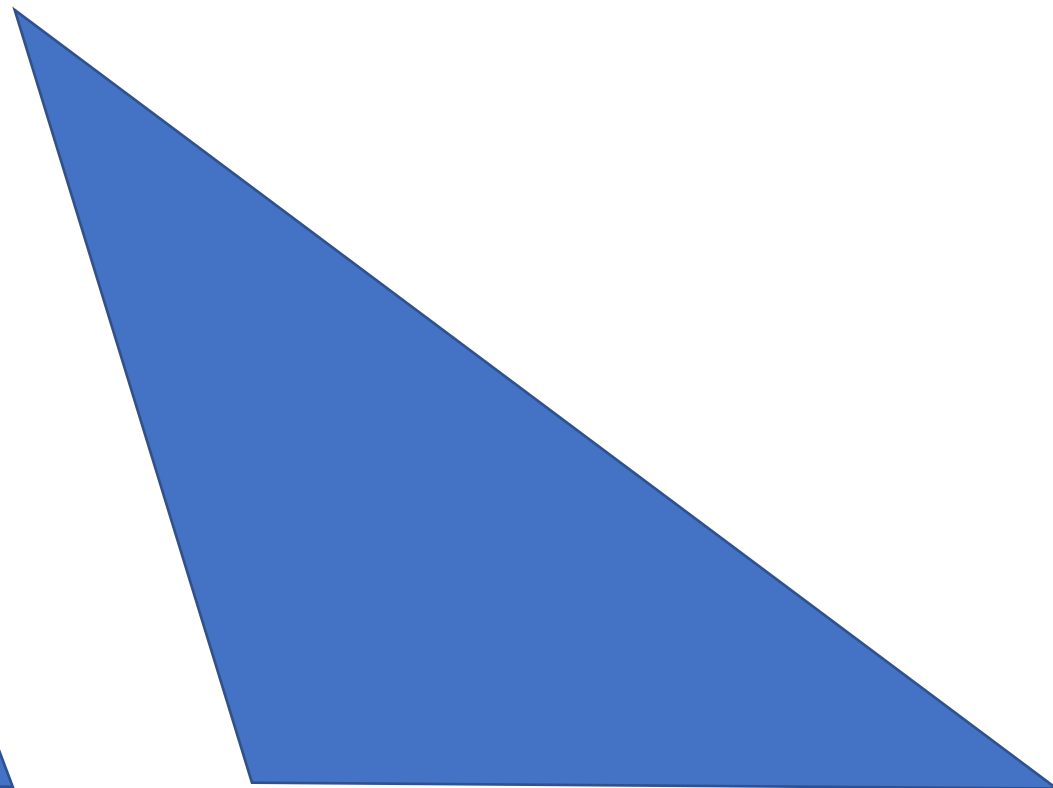
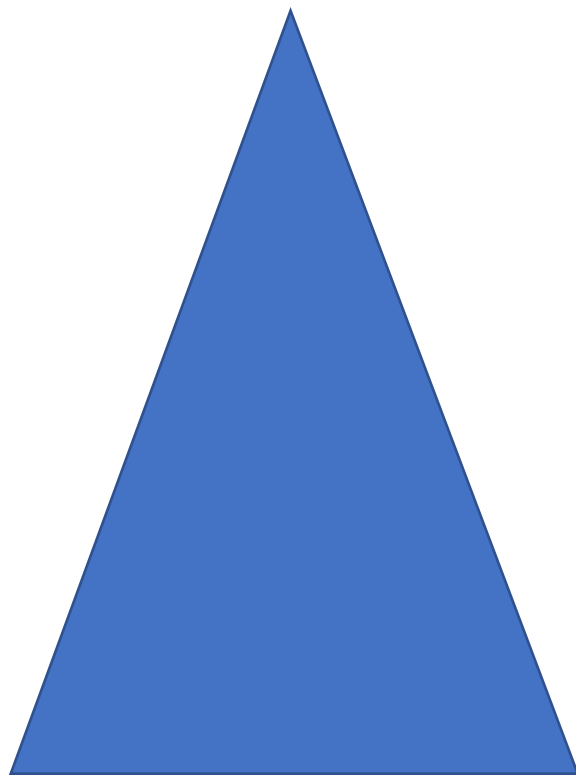


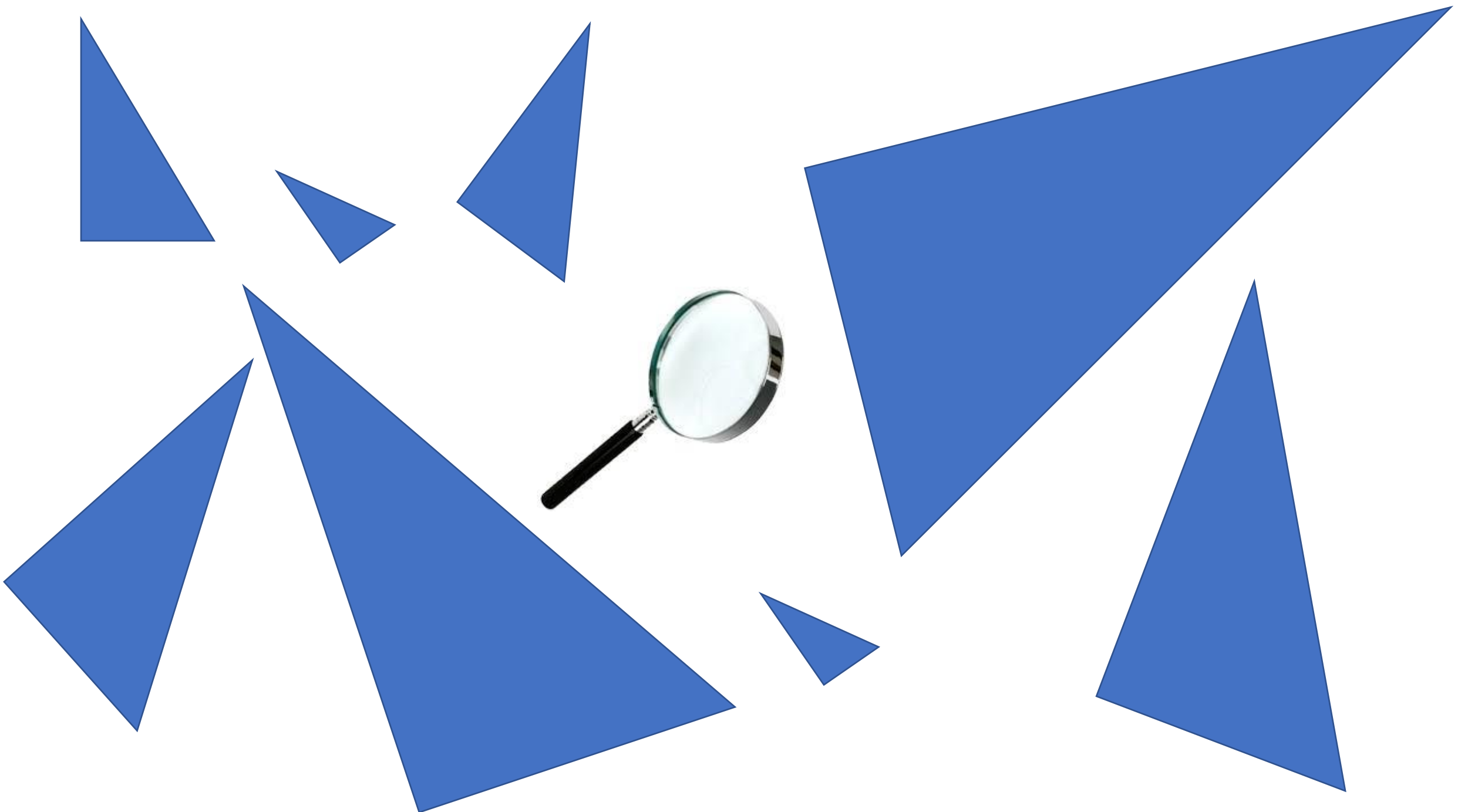
klinometr szkolny

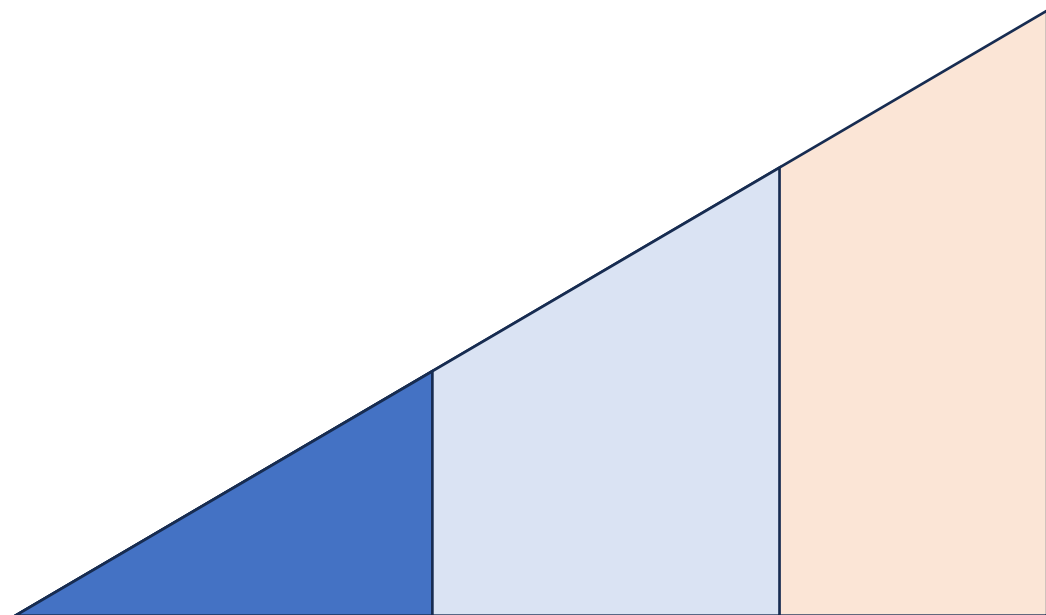




# Trójkąty różnych kształtów









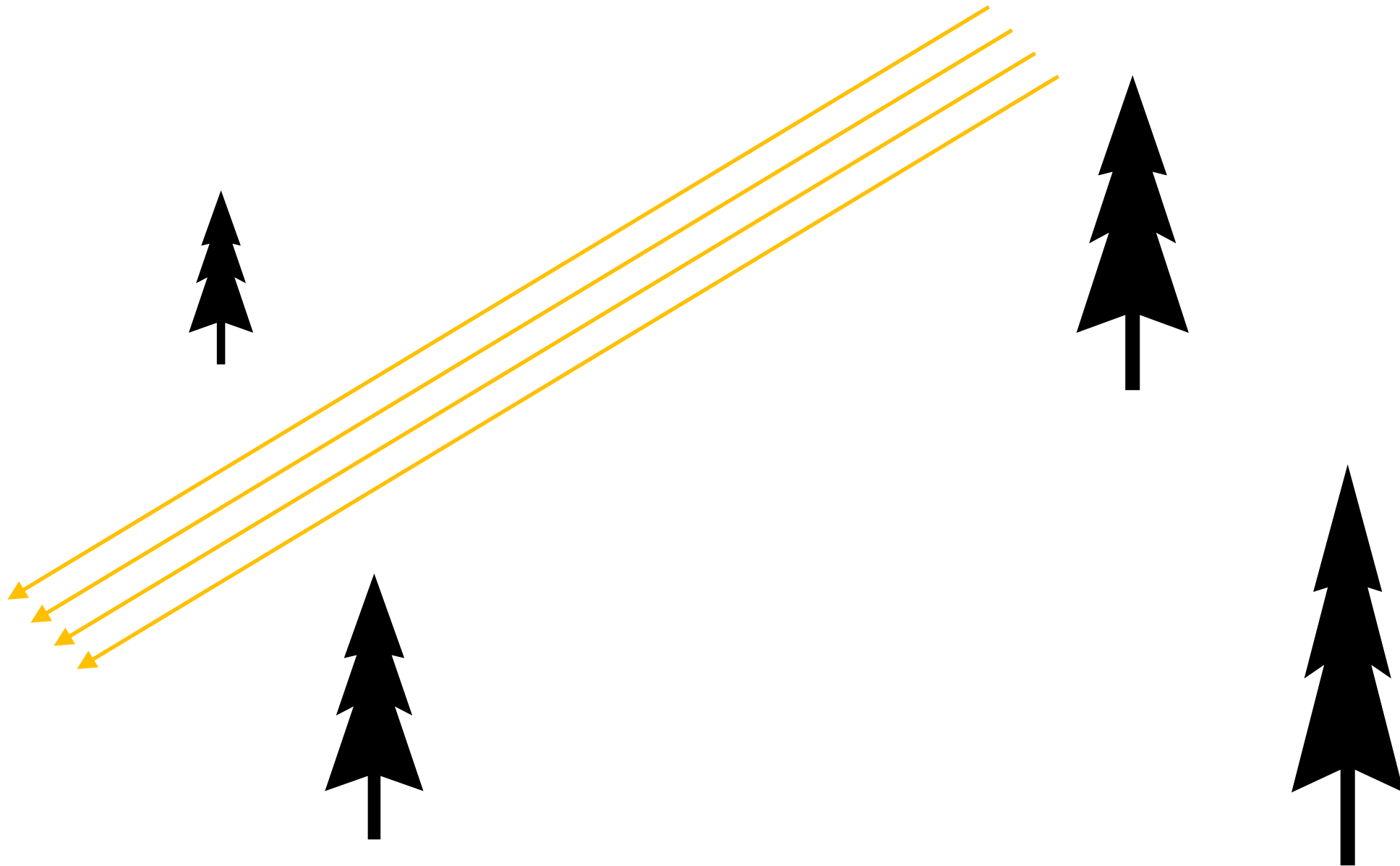
Trójkąt, który ma takie same kąty co inny trójkąt, ma też taki sam kształt.  
Trójkąty te różnią się tylko wielkością.  
Mówimy o nich, że są **podobne**.  
Boki jednego są powiększone tyle samo razy w stosunku do boków drugiego.  
To powiększenie nazywamy skalą podobieństwa.

**cecha podobieństwa trójkątów kkk**



# WYCIECZKA DO LASU





**A**

**60**

**36**

**C**

**54**

$54 : 60 = 0.9$   
 $36 \cdot 0.9 = 32.4$

**B**

**33**

**?**

$33 : 60 = 0.55$   
 $36 \cdot 0.55 = 19.8$

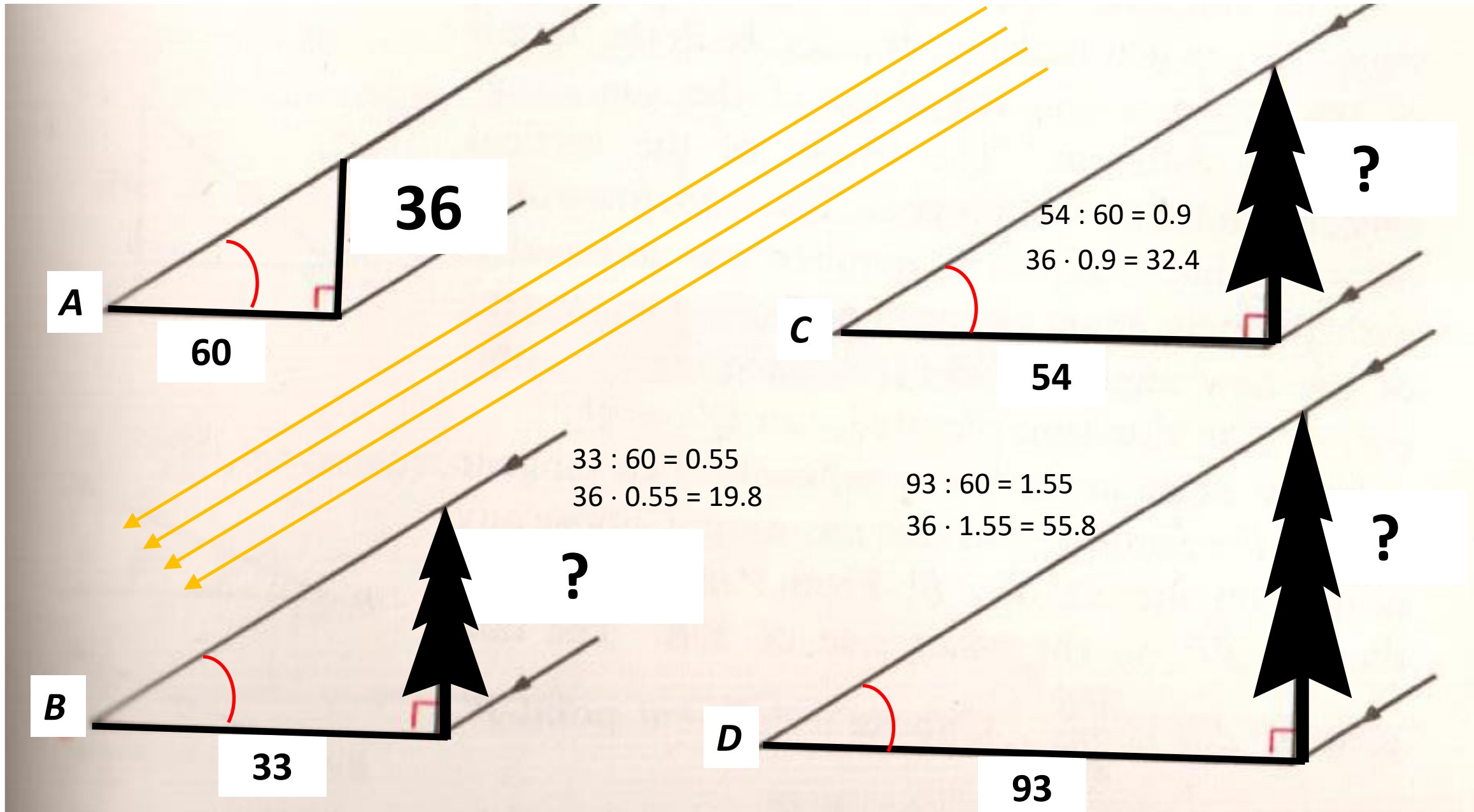
**D**

**93**

$93 : 60 = 1.55$   
 $36 \cdot 1.55 = 55.8$

**?**

**?**



**A**

**60**

**36**

**C**

**54**

**32.4**

**B**

**33**

**19.8**

**D**

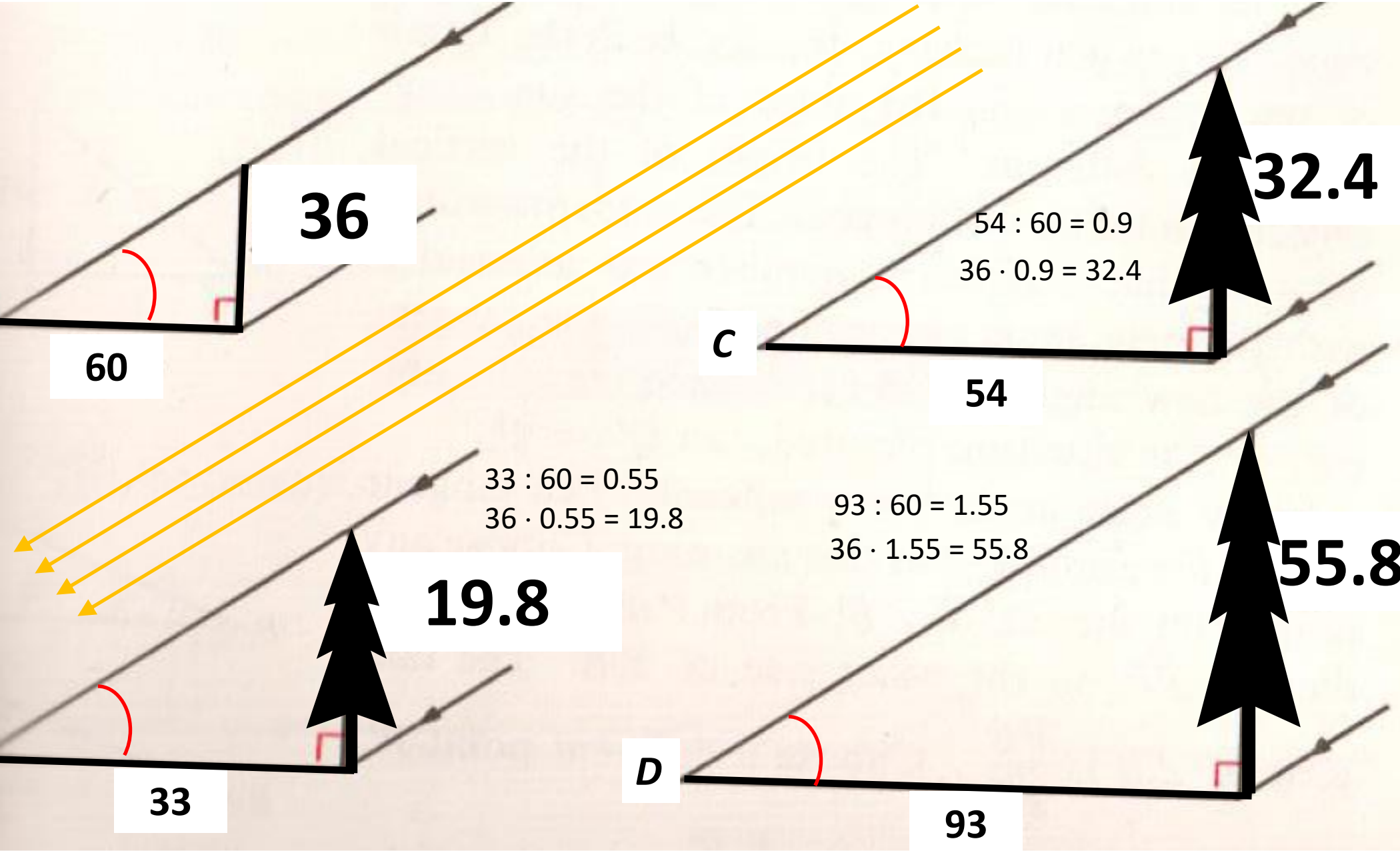
**93**

**55.8**

$54 : 60 = 0.9$   
 $36 \cdot 0.9 = 32.4$

$33 : 60 = 0.55$   
 $36 \cdot 0.55 = 19.8$

$93 : 60 = 1.55$   
 $36 \cdot 1.55 = 55.8$



**A**

$$60 \cdot 0.6 = 36$$

**60**

**36**

$$36/60 = 0.6$$

$$36 \cdot 0.9 / 60 \cdot 0.9 = 0.6$$

$$36 \cdot k / 60 \cdot k = 0.6$$

kāt A ↔ 0.6

**C**

$$54 \cdot 0.6 = 32,4$$

**54**

**32.4**

**B**

$$33 \cdot 0.6 = 19,8$$

**33**

**19.8**

**D**

$$93 \cdot 0.6 = 55.8$$

**93**

**55.8**

**A**

**60**

**36**

$$\begin{aligned}\tan A &= 36/60 = \\ &= 32.4/54 = \\ &= 19.8/33 = \\ &= 55.8/93 = \\ &= 0.6\end{aligned}$$

**C**

**54**

**32.4**

**B**

**33**

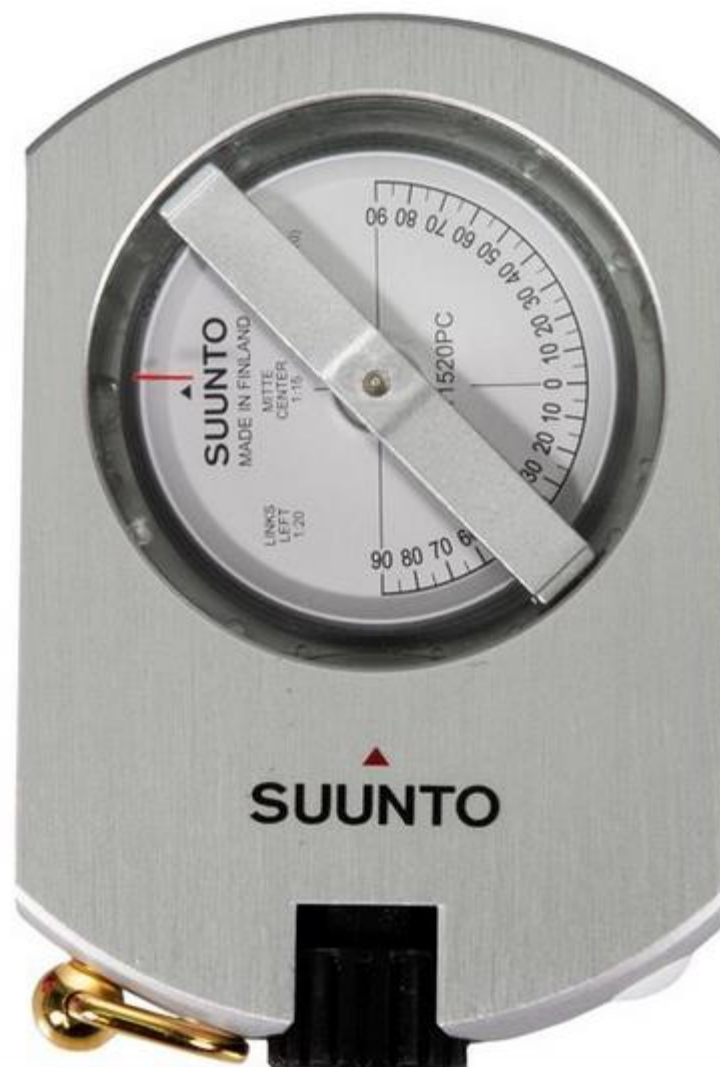
**19.8**

kāt A = ???

**D**

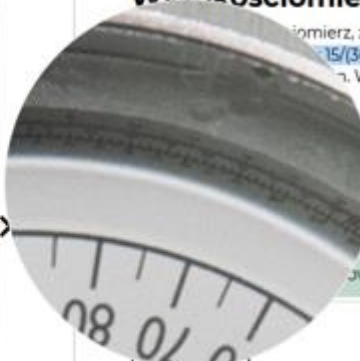
**93**

**55.8**



**Wysokościomierz:**

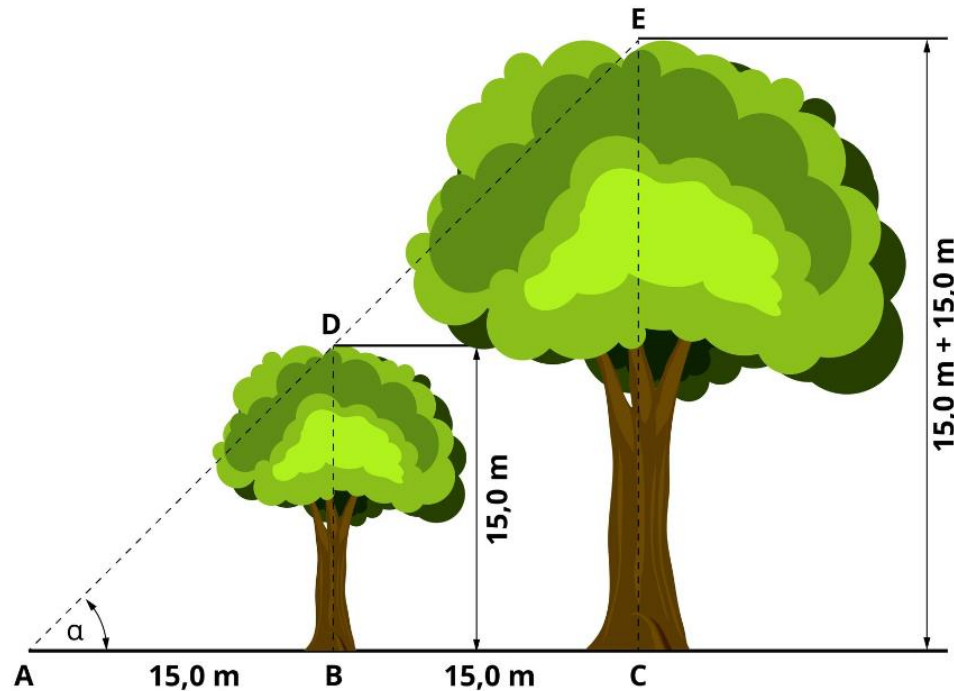
...mierz, zw  
...15/(30)  
...n. Wy



Cena z VAT Koszty wysyłki | Waga prz

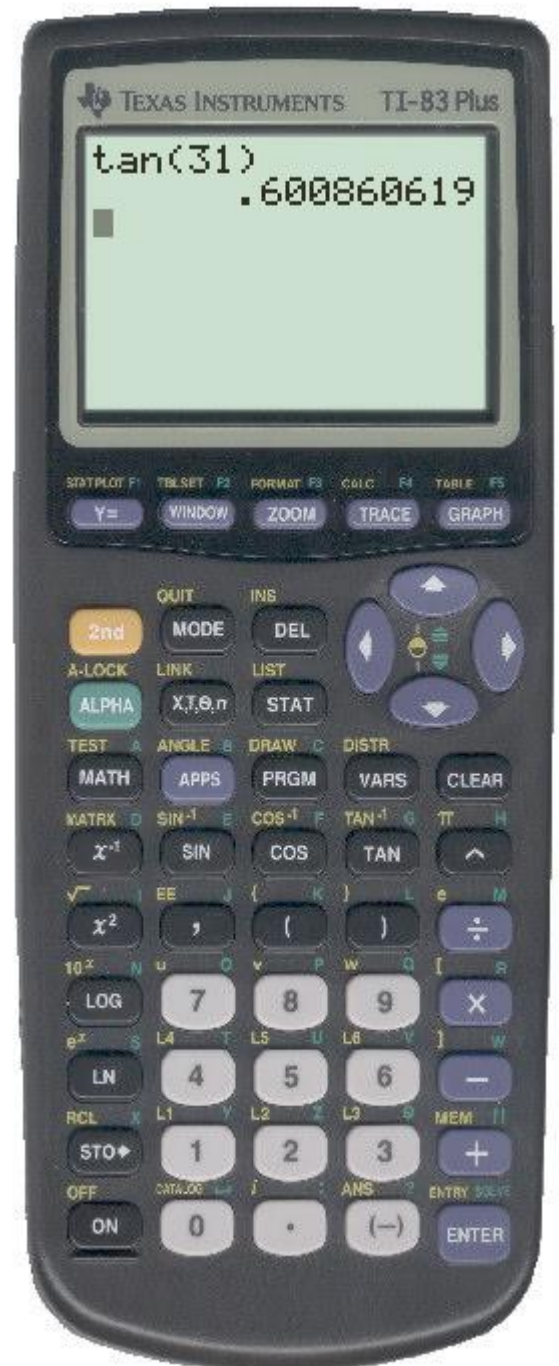
- 🚚 Dostawa w ciągu 1-3 dni rob
- 📍 Ten towar możesz odebrać  
Suchym Lesie k. Poznania,  
poinformujemy Cię w osob

Za pomocą wysokościomierza Suunto można dokonać pomiaru wysokości drzew z odległości 15 m i 20 m oraz ich wielokrotności. Zazwyczaj przyjmuje się odległość 30 m, a wyjątkowo 40 m. Przy założeniu, że nasz wzrok znajduje się na wysokości podstawy drzewa w odległości 15 m, wykonujemy pomiar, odczytując wysokość ze skali 1 : 15 wynoszącą 15 m. Jeśli w odległości 30 m jest drugie drzewo, jego wysokość odczytujemy z tej samej skali, ponieważ kąt  $\alpha$  się nie zmienił. Korzystając z podobieństwa trójkątów, można wówczas wykazać, że proporcja  $AB/BD$  jest taka sama jak  $AC/CE$ . Ze względu na to, że pomiar drugiego drzewa ( $CE$ ) wykonany został z odległości 30 m ( $15\text{ m} \times 2$ ), a odczyt był wykonany ze skali 1 : 15, odczyt z tej skali należy pomnożyć przez 2.



| KAT | TANGENS |
|-----|---------|
| 5°  | .087    |
| 10° | .176    |
| 15° | .268    |
| 20° | .364    |
| 25° | .466    |
| 30° | .577    |
| 35° | .700    |
| 40° | .839    |
| 45° | 1.000   |
| 50° | 1.192   |
| 55° | 1.428   |
| 60° | 1.732   |
| 65° | 2.145   |
| 70° | 2.747   |
| 75° | 3.732   |
| 80° | 5.671   |
| 85° | 11.430  |

| KAT | TANGENS |
|-----|---------|
| 30° | 0.58    |
| 31° | 0.60    |
| 32° | 0.62    |
| 33° | 0.65    |
| 34° | 0.67    |
| 35° | 0.70    |
| 36° | 0.73    |



**A**

**60**

**36**

$$\begin{aligned}\tan A &= 36/60 = \\ &= 32.4/54 = \\ &= 19.8/33 = \\ &= 55.8/93 = \\ &= 0.6\end{aligned}$$

**C**

**54**

**32.4**

kāt A = 31°

**B**

**33**

**19.8**

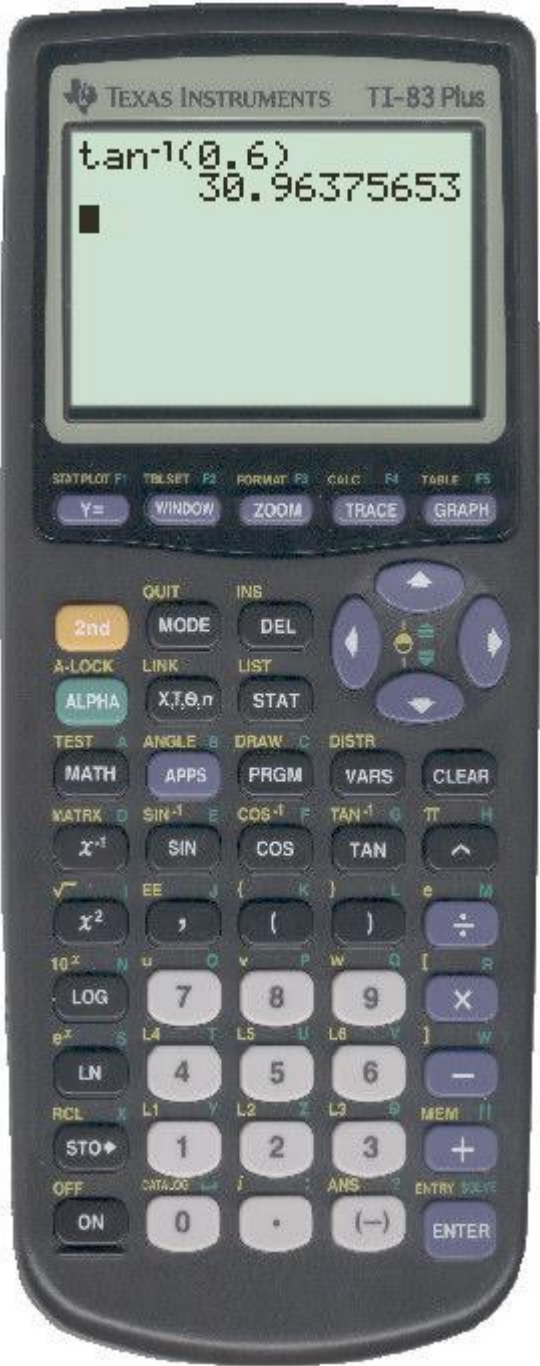
**D**

**93**

**55.8**

KĄT SIN COS TAN KĄT SIN COS TAN

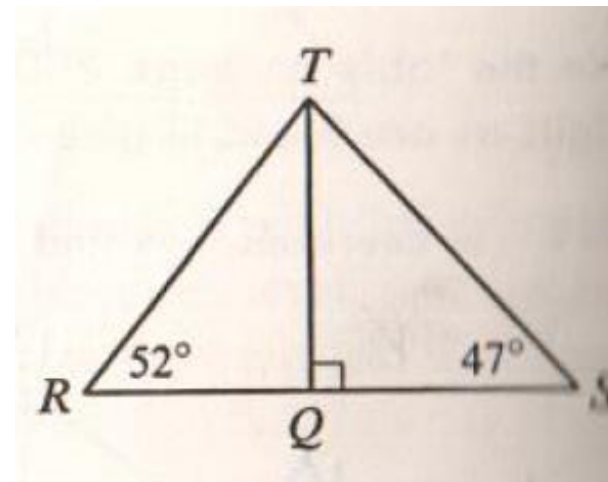
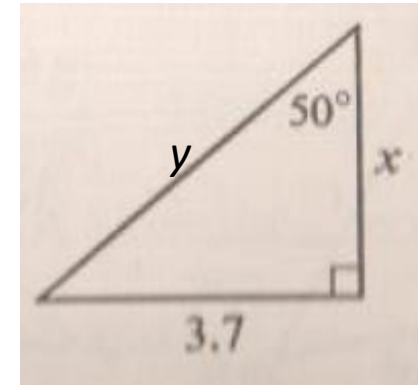
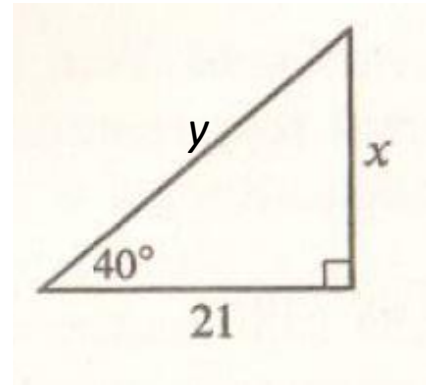
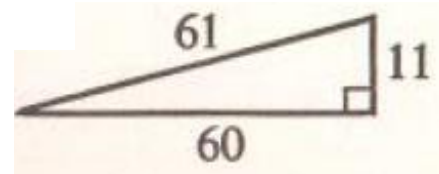
|     |       |       |        |     |       |       |         |
|-----|-------|-------|--------|-----|-------|-------|---------|
| 1°  | .0175 | .9998 | .0175  | 46° | .7193 | .6947 | 1.0355  |
| 2°  | .0349 | .9994 | .0349  | 47° | .7314 | .6820 | 1.0724  |
| 3°  | .0523 | .9986 | .0524  | 48° | .7431 | .6691 | 1.1106  |
| 4°  | .0698 | .9976 | .0699  | 49° | .7547 | .6561 | 1.1504  |
| 5°  | .0872 | .9962 | .0875  | 50° | .7660 | .6428 | 1.1918  |
| 6°  | .1045 | .9945 | .1051  | 51° | .7771 | .6293 | 1.2349  |
| 7°  | .1219 | .9925 | .1228  | 52° | .7880 | .6157 | 1.2799  |
| 8°  | .1392 | .9903 | .1405  | 53° | .7986 | .6018 | 1.3270  |
| 9°  | .1564 | .9877 | .1584  | 54° | .8090 | .5878 | 1.3764  |
| 10° | .1736 | .9848 | .1763  | 55° | .8192 | .5736 | 1.4281  |
| 11° | .1908 | .9816 | .1944  | 56° | .8290 | .5592 | 1.4826  |
| 12° | .2079 | .9781 | .2126  | 57° | .8387 | .5446 | 1.5399  |
| 13° | .2250 | .9744 | .2309  | 58° | .8480 | .5299 | 1.6003  |
| 14° | .2419 | .9703 | .2493  | 59° | .8572 | .5150 | 1.6643  |
| 15° | .2588 | .9659 | .2679  | 60° | .8660 | .5000 | 1.7321  |
| 16° | .2756 | .9613 | .2867  | 61° | .8746 | .4848 | 1.8040  |
| 17° | .2924 | .9563 | .3057  | 62° | .8829 | .4695 | 1.8807  |
| 18° | .3090 | .9511 | .3249  | 63° | .8910 | .4540 | 1.9626  |
| 19° | .3256 | .9455 | .3443  | 64° | .8988 | .4384 | 2.0503  |
| 20° | .3420 | .9397 | .3640  | 65° | .9063 | .4226 | 2.1445  |
| 21° | .3584 | .9336 | .3839  | 66° | .9135 | .4067 | 2.2460  |
| 22° | .3746 | .9272 | .4040  | 67° | .9205 | .3907 | 2.3559  |
| 23° | .3907 | .9205 | .4245  | 68° | .9272 | .3746 | 2.4751  |
| 24° | .4067 | .9135 | .4452  | 69° | .9336 | .3584 | 2.6051  |
| 25° | .4226 | .9063 | .4663  | 70° | .9397 | .3420 | 2.7475  |
| 26° | .4384 | .8988 | .4877  | 71° | .9455 | .3256 | 2.9042  |
| 27° | .4540 | .8910 | .5095  | 72° | .9511 | .3090 | 3.0777  |
| 28° | .4695 | .8829 | .5317  | 73° | .9563 | .2924 | 3.2709  |
| 29° | .4848 | .8746 | .5543  | 74° | .9613 | .2756 | 3.4874  |
| 30° | .5000 | .8660 | .5774  | 75° | .9659 | .2588 | 3.7321  |
| 31° | .5150 | .8572 | .6009  | 76° | .9703 | .2419 | 4.0108  |
| 32° | .5299 | .8480 | .6249  | 77° | .9744 | .2250 | 4.3315  |
| 33° | .5446 | .8387 | .6494  | 78° | .9781 | .2079 | 4.7046  |
| 34° | .5592 | .8290 | .6745  | 79° | .9816 | .1908 | 5.1446  |
| 35° | .5736 | .8192 | .7002  | 80° | .9848 | .1736 | 5.6713  |
| 36° | .5878 | .8090 | .7265  | 81° | .9877 | .1564 | 6.3138  |
| 37° | .6018 | .7986 | .7536  | 82° | .9903 | .1392 | 7.1154  |
| 38° | .6157 | .7880 | .7813  | 83° | .9925 | .1219 | 8.1443  |
| 39° | .6293 | .7771 | .8098  | 84° | .9945 | .1045 | 9.5144  |
| 40° | .6428 | .7660 | .8391  | 85° | .9962 | .0872 | 11.4301 |
| 41° | .6561 | .7547 | .8693  | 86° | .9976 | .0698 | 14.3007 |
| 42° | .6691 | .7431 | .9004  | 87° | .9986 | .0523 | 19.0811 |
| 43° | .6820 | .7314 | .9325  | 88° | .9994 | .0349 | 28.6363 |
| 44° | .6947 | .7193 | .9657  | 89° | .9998 | .0175 | 57.2900 |
| 45° | .7071 | .7071 | 1.0000 |     |       |       |         |



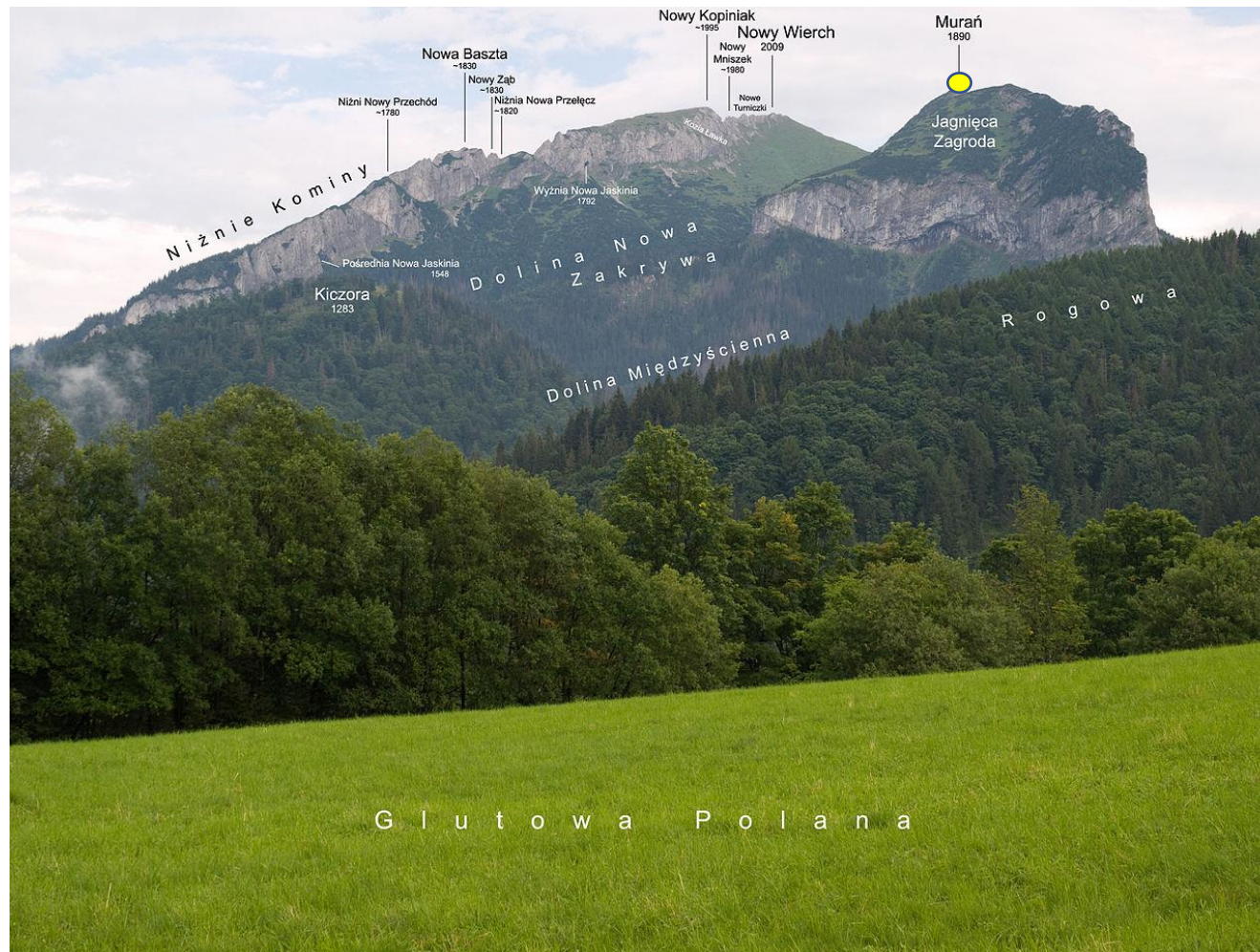
# ĆWICZENIA

# ĆWICZENIA

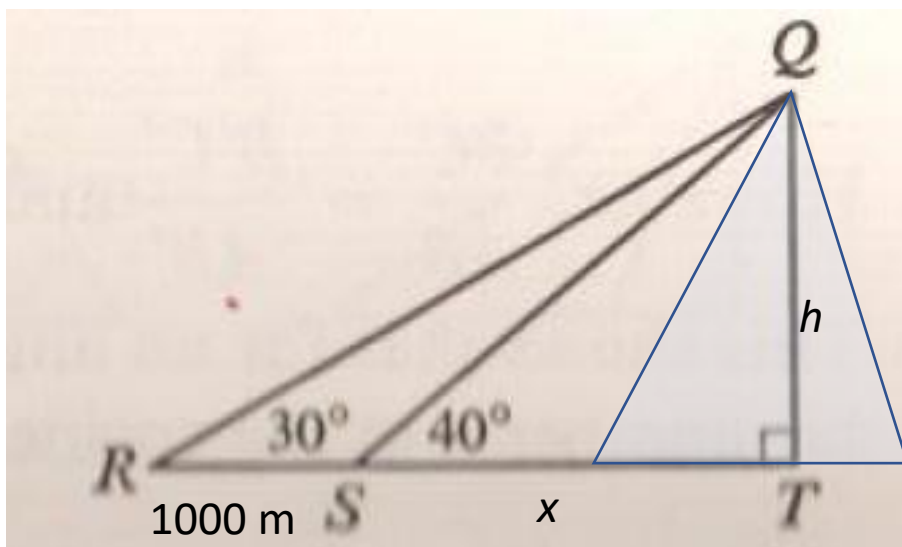
1. Oblicz tangensy, sinusy i kosinusy kątów ostrych trójkąta z rys. 1.
2. Podaj miary kątów poprzedniego trójkąta z dokładnością do  $5^\circ$ .
3. Oblicz długości boków  $x$  i  $y$  z rys. 2 i 3.
4. Odcinek  $TQ$  ma 13.2 cm. Znajdź długość  $RS$  (rys. 4).
5. Odcinek  $RT$  ma długość 127. Znajdź długość  $ST$  (rys. 4).
6. Podaj (z głowy) wartość wyrażenia  $\tan 30^\circ \cdot \tan 60^\circ$ .



# Jak wysoki jest Murań?



# Jak wysoki jest Murań?



$$h/x = \tan 40^\circ = 84/100$$

$$x/h = 100/84$$

$$h/(x+1000) = \tan 30^\circ = 58/100$$

$$(x+1000)/h = 100/58$$

$$(x+1000)/h = x/h + 1000/h = 100/84 + 1000/h = 100/58$$

$$1000/h = 100/58 - 100/84 = 1,72 - 1,19 = 53/100$$

$$h/1000 = 100/53 = 1.89$$

$$h = 1890 \text{ m}$$

# Jak duże jest Słońce?

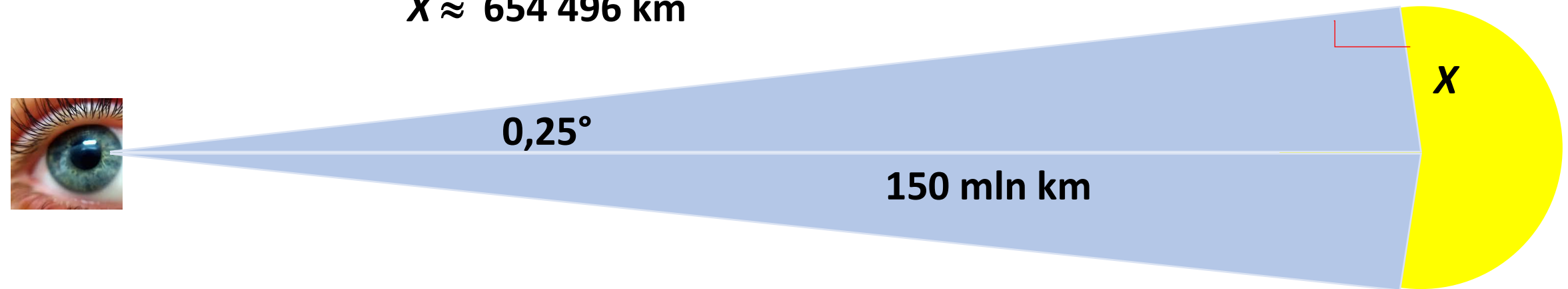
$$X : 150\,000\,000 = \sin 0.25^\circ$$

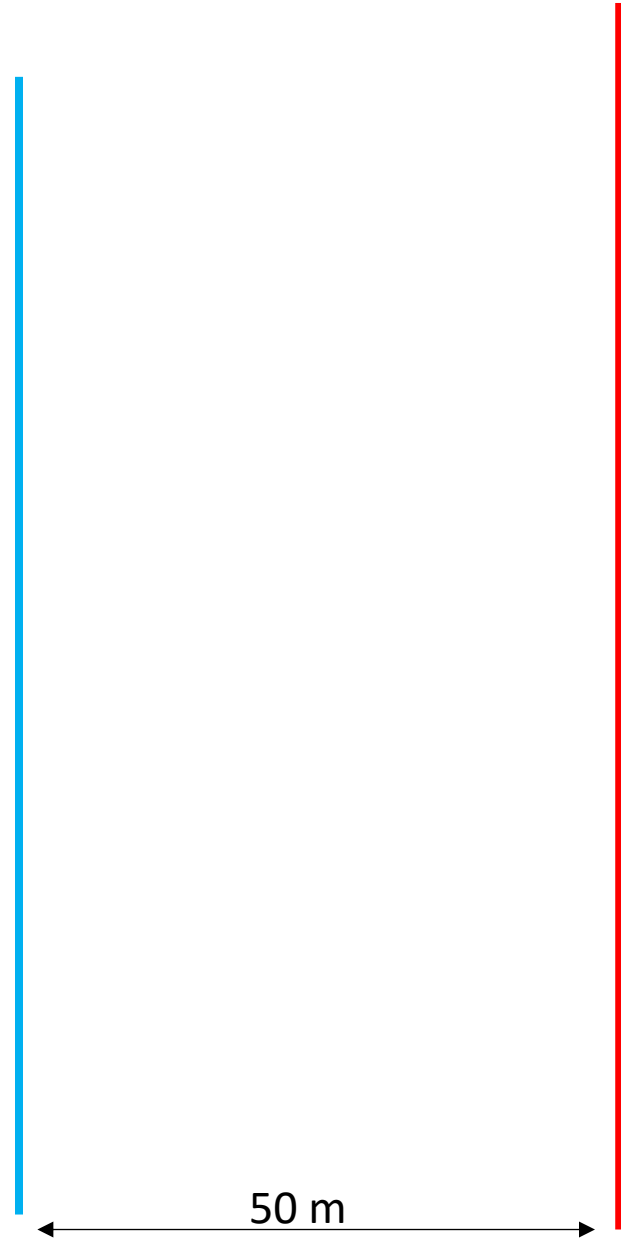
$$\sin 0.25^\circ = 0.004$$

$$X : 150\,000\,000 = 0.004$$

$$X = 0.004 \cdot 150\,000\,000$$

$$X \approx 654\,496 \text{ km}$$





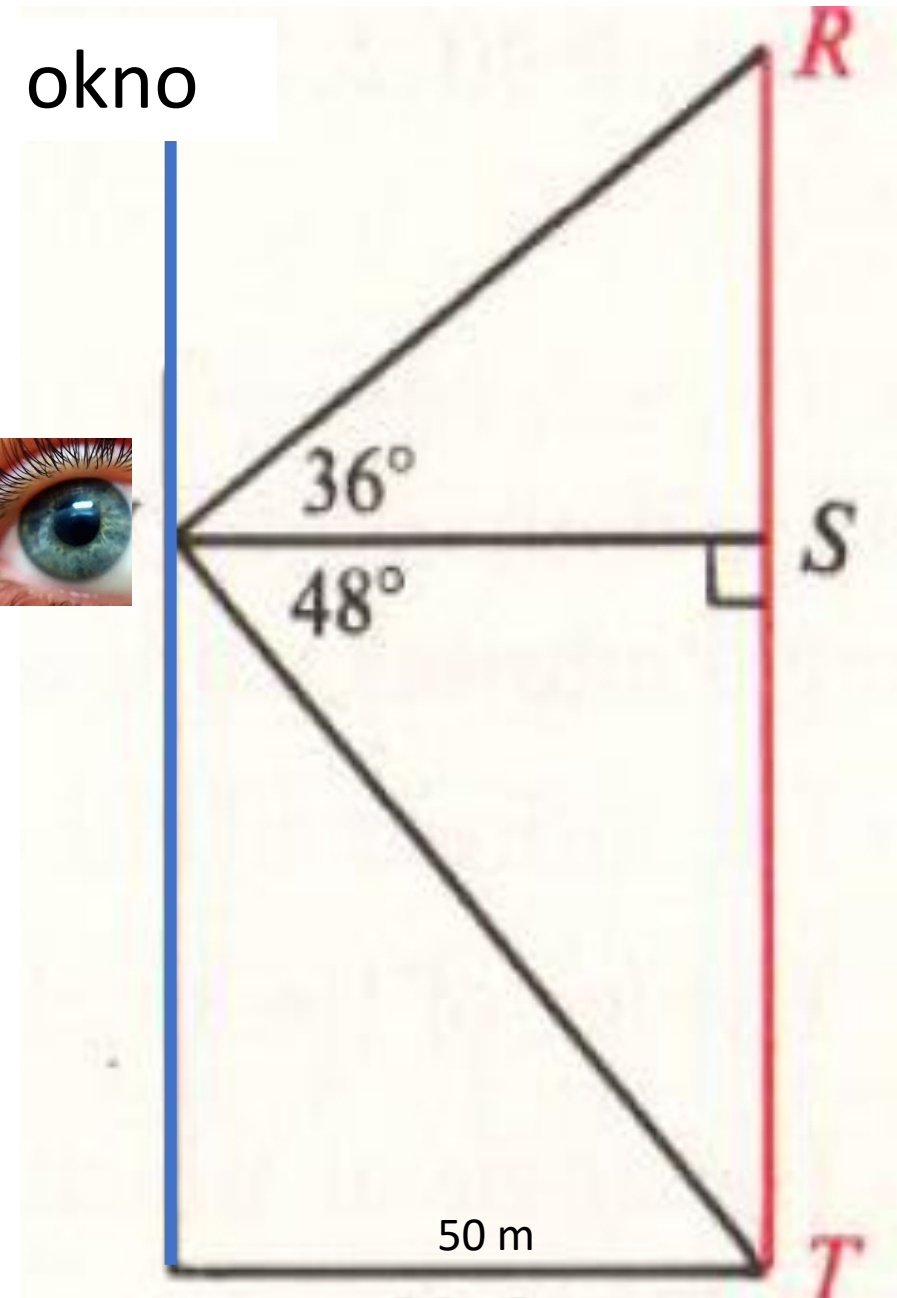
$$ST/50 = \tan 48^\circ \quad ST = 50 \cdot 1.11 = 55.5$$

$$RS/50 = \tan 36^\circ \quad RS = 50 \cdot 0.73 = 36.5$$

93 m

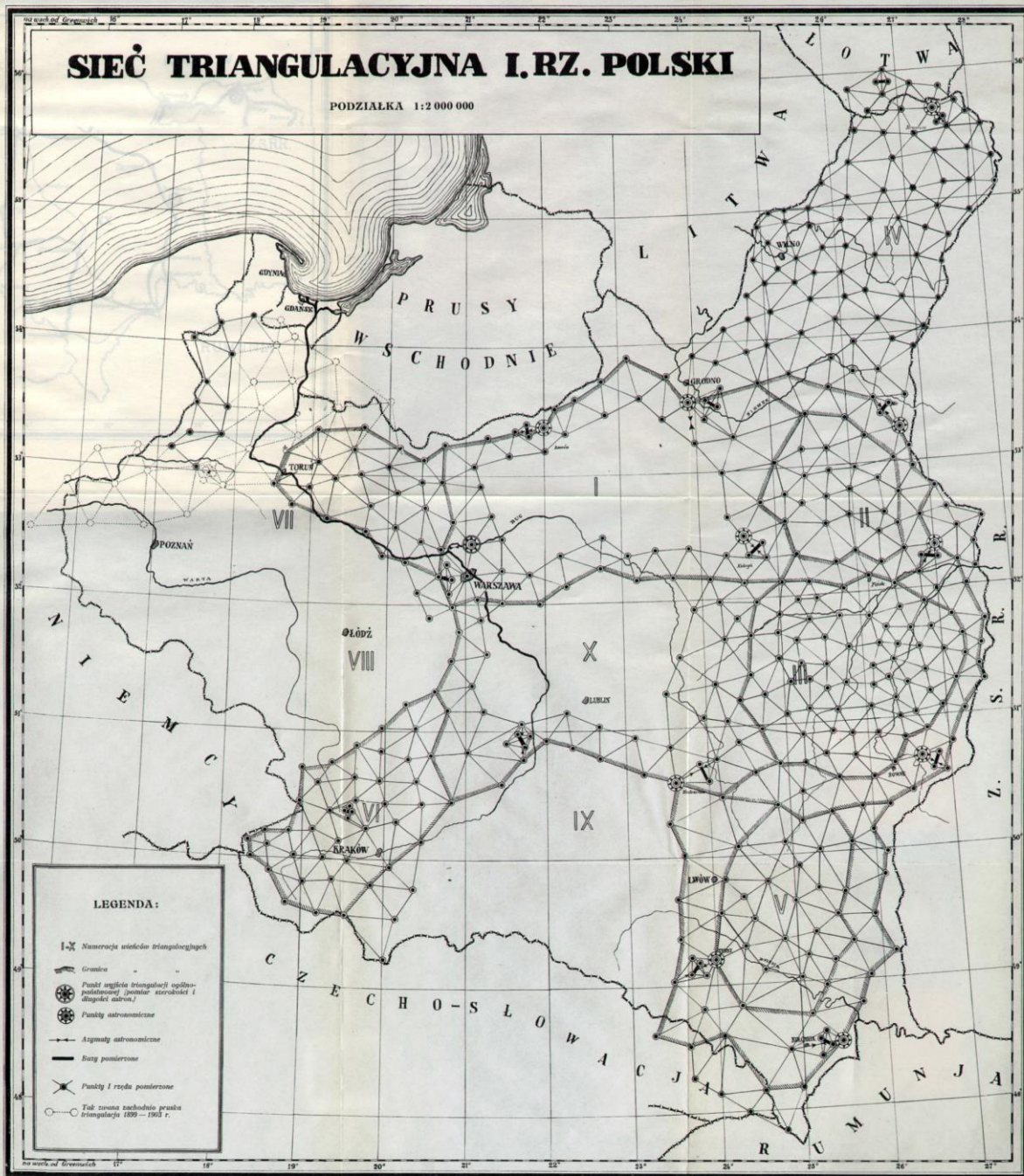


okno

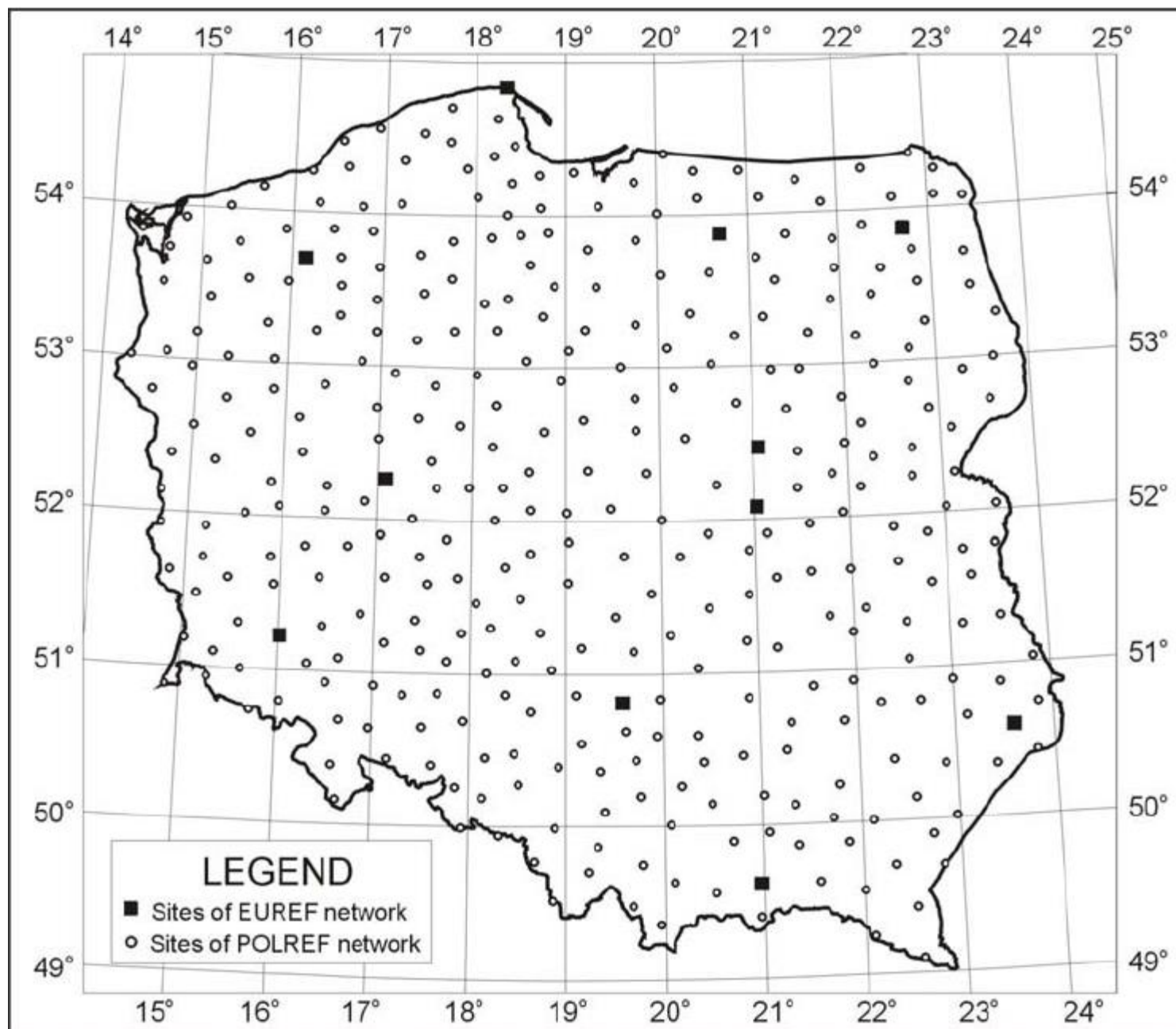


I TO NIE BYŁA  
TRYGONOMETRIA

A CO JEST  
TRYGONOMETRIA?



**Triangulacja** w geodezji to metoda pomiaru odległości polegająca na precyzyjnym pomiarze kątów między wszystkimi sąsiednimi punktami oraz długości co najmniej jednego boku w sieci składającej się z trójkątów. Taka sieć nazywana jest siecią triangulacyjną.



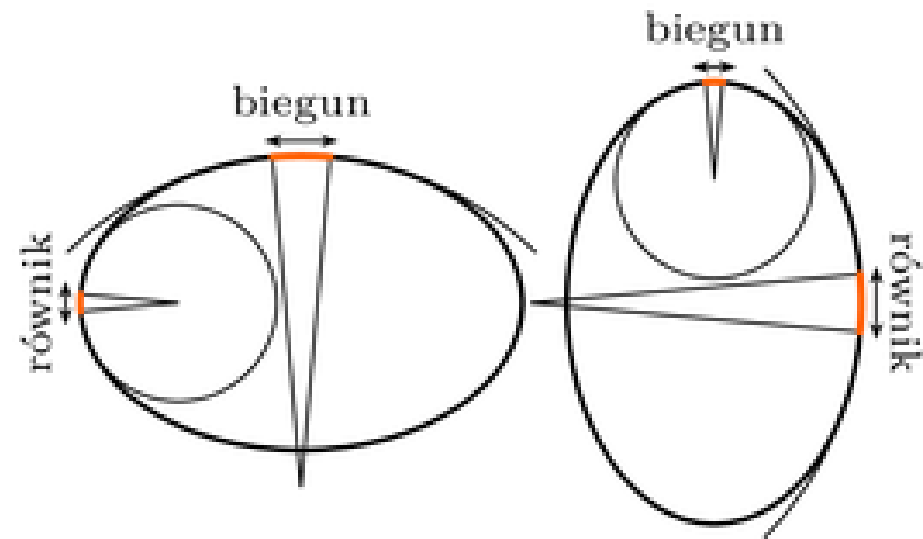
# południk Struvego



## Wyprawa Maupertiusa do Laponii



Pierre Louis Moreau de Maupertuis  
(1698–1759)



I I I.

*Calcul des Triangles de la première suite.**A C H.**Angles observés, réduits à l'horison.    Angles corrigés pour le calcul.*

Fig. 2.

|               |      |     |       |       |      |     |     |
|---------------|------|-----|-------|-------|------|-----|-----|
| <i>CAH...</i> | 112° | 21' | 32,9" | ...   | 112° | 21' | 17" |
| <i>ACH...</i> | 30   | 56  | 53,4  | ...   | 30   | 56  | 47  |
| <i>AHC...</i> | 36   | 42  | 3,1   | ...   | 36   | 41  | 56  |
| <hr/>         |      |     |       | <hr/> |      |     |     |
|               | 180  | 0   | 29,4  |       | 180  | 0   | 0   |

*CHK.*

|               |     |    |      |       |     |    |    |
|---------------|-----|----|------|-------|-----|----|----|
| <i>CHK...</i> | 36  | 4  | 54,7 | ...   | 36  | 4  | 46 |
| <i>CKH...</i> | 43  | 45 | 35,6 | ...   | 43  | 45 | 26 |
| <i>KCH...</i> | 100 | 9  | 56,8 | ...   | 100 | 9  | 48 |
| <hr/>         |     |    |      | <hr/> |     |    |    |
|               | 180 | 0  | 27,1 |       | 180 | 0  | 0  |

*CKT.*

|               |     |    |      |       |     |    |    |
|---------------|-----|----|------|-------|-----|----|----|
| <i>KCT...</i> | 37  | 9  | 12,0 | ...   | 37  | 9  | 7  |
| <i>CKT...</i> | 118 | 28 | 12,0 | ...   | 118 | 28 | 3  |
| <i>CTK...</i> | 24  | 22 | 54,3 | ...   | 24  | 22 | 50 |
| <hr/>         |     |    |      | <hr/> |     |    |    |
|               | 180 | 0  | 18,3 |       | 180 | 0  | 0  |

*AHP.*

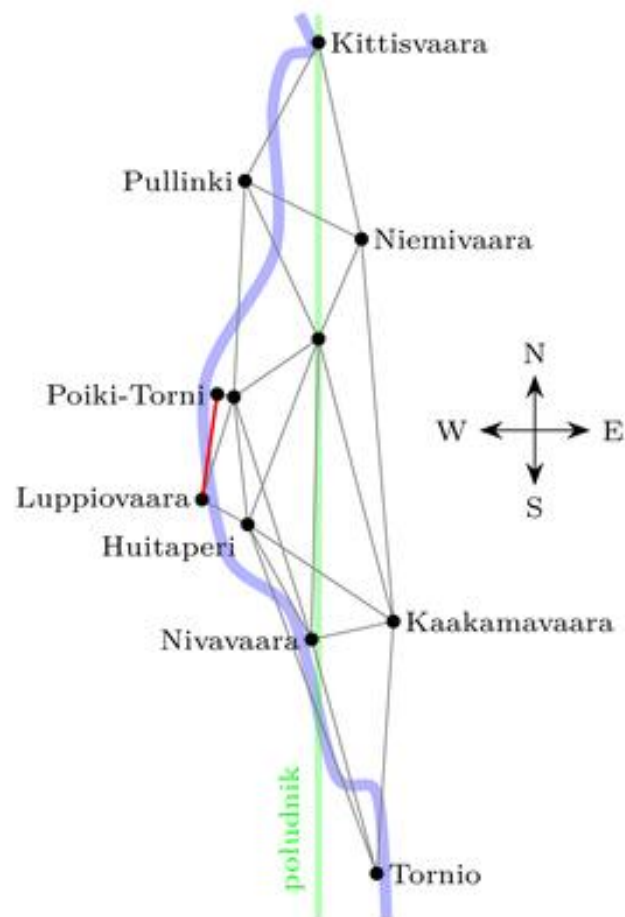
|               |     |    |      |       |     |    |    |
|---------------|-----|----|------|-------|-----|----|----|
| <i>AHP...</i> | 94  | 53 | 49,7 | ...   | 94  | 53 | 56 |
| <i>HAP...</i> | 53  | 45 | 56,7 | ...   | 53  | 46 | 3  |
| <i>APH...</i> | 31  | 19 | 55,5 | ...   | 31  | 20 | 1  |
| <hr/>         |     |    |      | <hr/> |     |    |    |
|               | 179 | 59 | 41,9 |       | 180 | 0  | 0  |

*HNP.*

|               |     |    |      |       |     |    |    |
|---------------|-----|----|------|-------|-----|----|----|
| <i>HNP...</i> | 93  | 25 | 7,5  | ...   | 93  | 25 | 1  |
| <i>NHP...</i> | 49  | 13 | 9,3  | ...   | 49  | 13 | 3  |
| <i>HPN...</i> | 37  | 22 | 2,1  | ...   | 37  | 21 | 56 |
| <hr/>         |     |    |      | <hr/> |     |    |    |
|               | 180 | 0  | 18,9 |       | 180 | 0  | 0  |

*NPQ.*

|               |     |    |      |       |     |    |    |
|---------------|-----|----|------|-------|-----|----|----|
| <i>NPQ...</i> | 87  | 52 | 24,3 | ...   | 87  | 52 | 17 |
| <i>NQP...</i> | 40  | 14 | 52,7 | ...   | 40  | 14 | 46 |
| <i>PNQ...</i> | 51  | 53 | 4,3  | ...   | 51  | 52 | 57 |
| <hr/>         |     |    |      | <hr/> |     |    |    |
|               | 180 | 0  | 21,3 |       | 180 | 0  | 0  |



# deltam

MATEMATYKA – FIZYKA – ASTRONOMIA – INFORMATYKA

ISSN 1663-0073  
p-issn 1663-0073  
t-issn 1663-0073

NR 6 (601) 2024

CENA 8 ZŁ VAT 8%  
PL ISSN 0137-3005 I NR IND 35 550 X  
MIESIĘCZNIK

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prawdziwego  
kształtu Ziemi  
str. 13



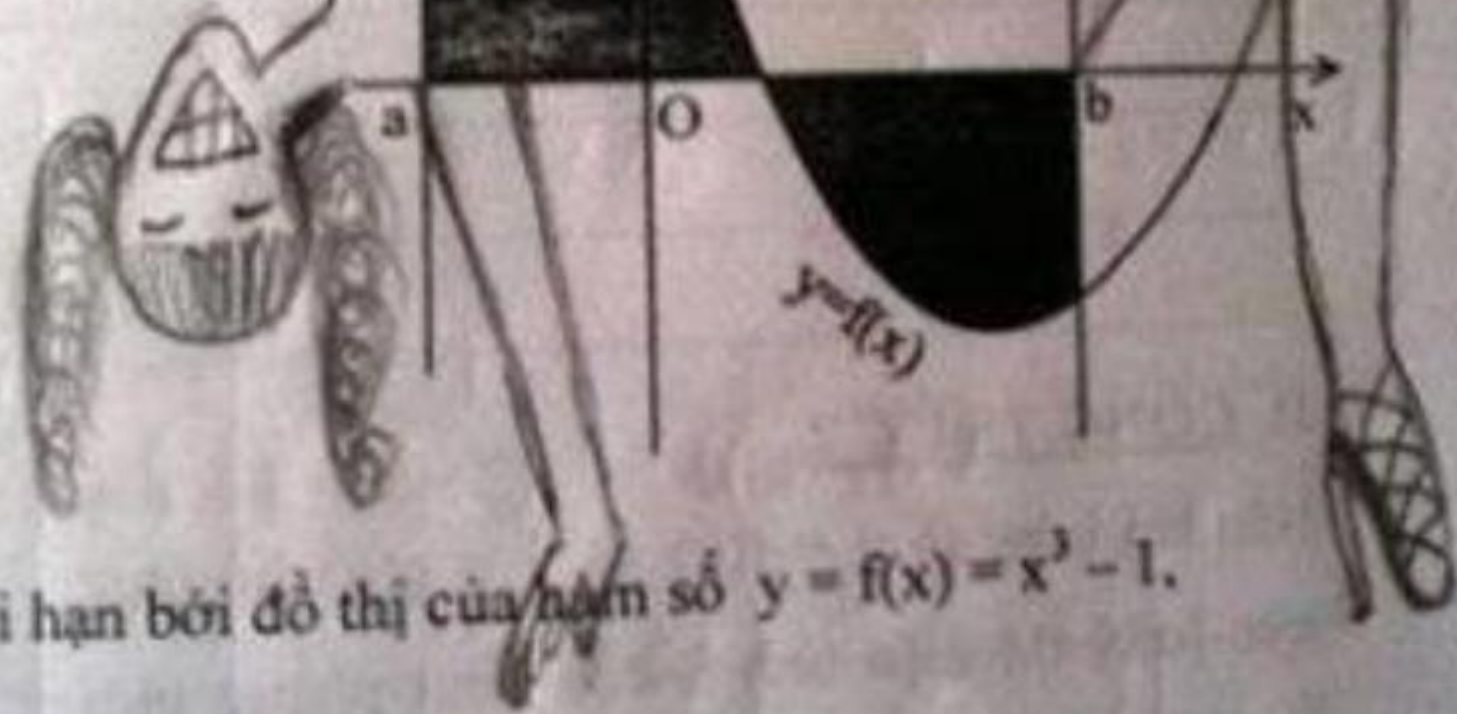
Nakład: 2800 egz.

I TO NIE BYŁA  
TRYGONOMETRIA

A CO JEST  
TRYGONOMETRIA?

N ĐỀ TÀI

diện tích  $S$  của  
 $f(x)$ , trục hoành



hạn bởi đồ thị của hàm số  $y = f(x) = x^3 - 1$ .