

Collaboration in the Mathematics Classroom

From Japan to Italy

About me



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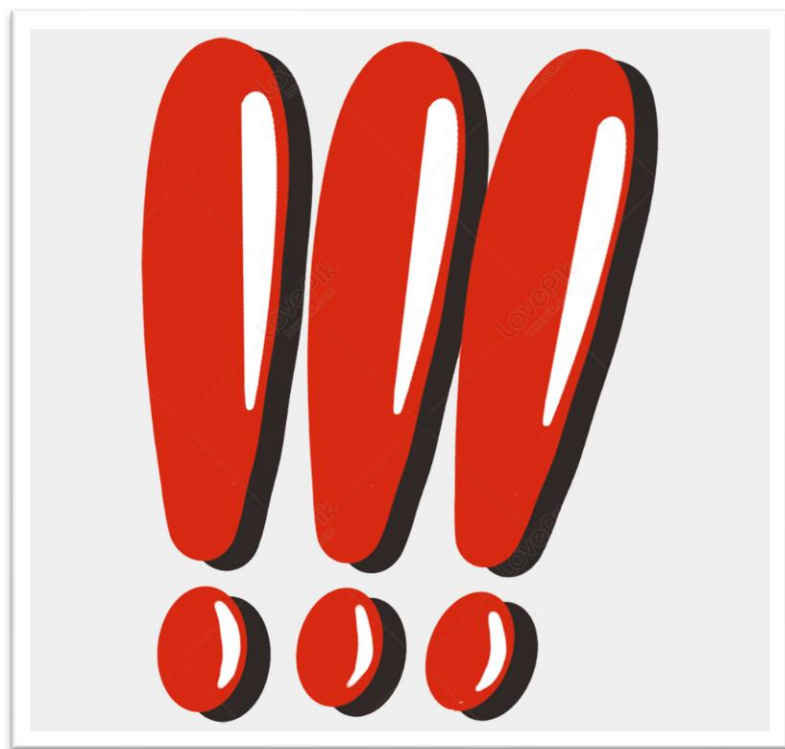
Teaching & Learning in collaborative environment



As a mathematics researcher, I believe that knowledge is socially constructed.

Mathematical understanding grows through dialogue, collaboration, and the collective refinement of ideas, not in isolation, but within a community of thinkers.

Some “products” from this field



For the pupils:

- Laboratory of Mathematics

For the teachers:

- Lesson Study
- Scuole Secondarie Potenziate in Matematica

The Mathematics Laboratory

based on Giacardi (2011)

Giovanni Vailati (1863 - 1909)

- Mathematician
- Historian
- Philosopher



Worked on the reform on *pre-fascism* Italian school:

[School is a laboratory,] a place where pupils are given the means to train themselves, under the guidance and advice of their teachers, to experiment and solve problems, [...] to test themselves in the face of obstacles and difficulties designed to challenge their ingenuity and cultivate their initiative.



Meraner Vorschlägen (1905)

- Redefinition of boundaries of secondary and higher teaching
- Functional thinking
- Application of mathematics
- Modelling
- Spatial intuition
- History and epistemology
- Connection with real problems and physics



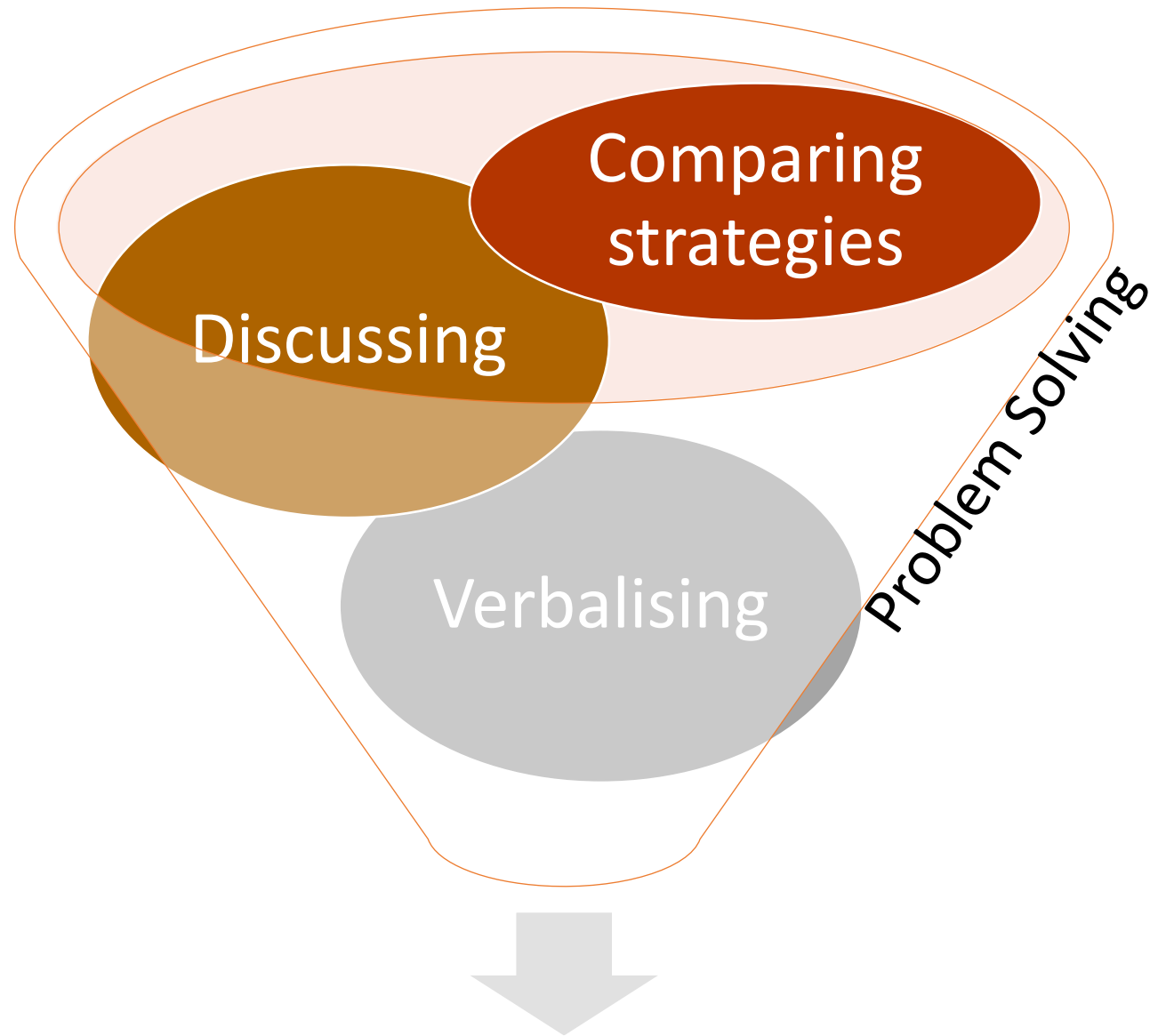
The «modern» laboratory of mathematics

The mathematics laboratory is not a physical place separate from the classroom, but rather a structured set of activities aimed at **constructing the meanings of mathematical objects**. The laboratory therefore involves people (students and teachers), structures (classrooms, tools, organisation of space and time), and ideas (projects, teaching activity plans, experiments).

Mathematics as a tool for citizenship

- which allows the subject to return to what has been done, seen (etc.), producing interpretations, explanations, answers to questions such as “why is it so?”
- which allows the subject to anticipate events, situations, etc., producing predictions, hypothetical discussions about possible worlds, answers to questions such as “what will it be like?”, “what could it be like?”

(from *Matematica per il Cittadino 2021*)



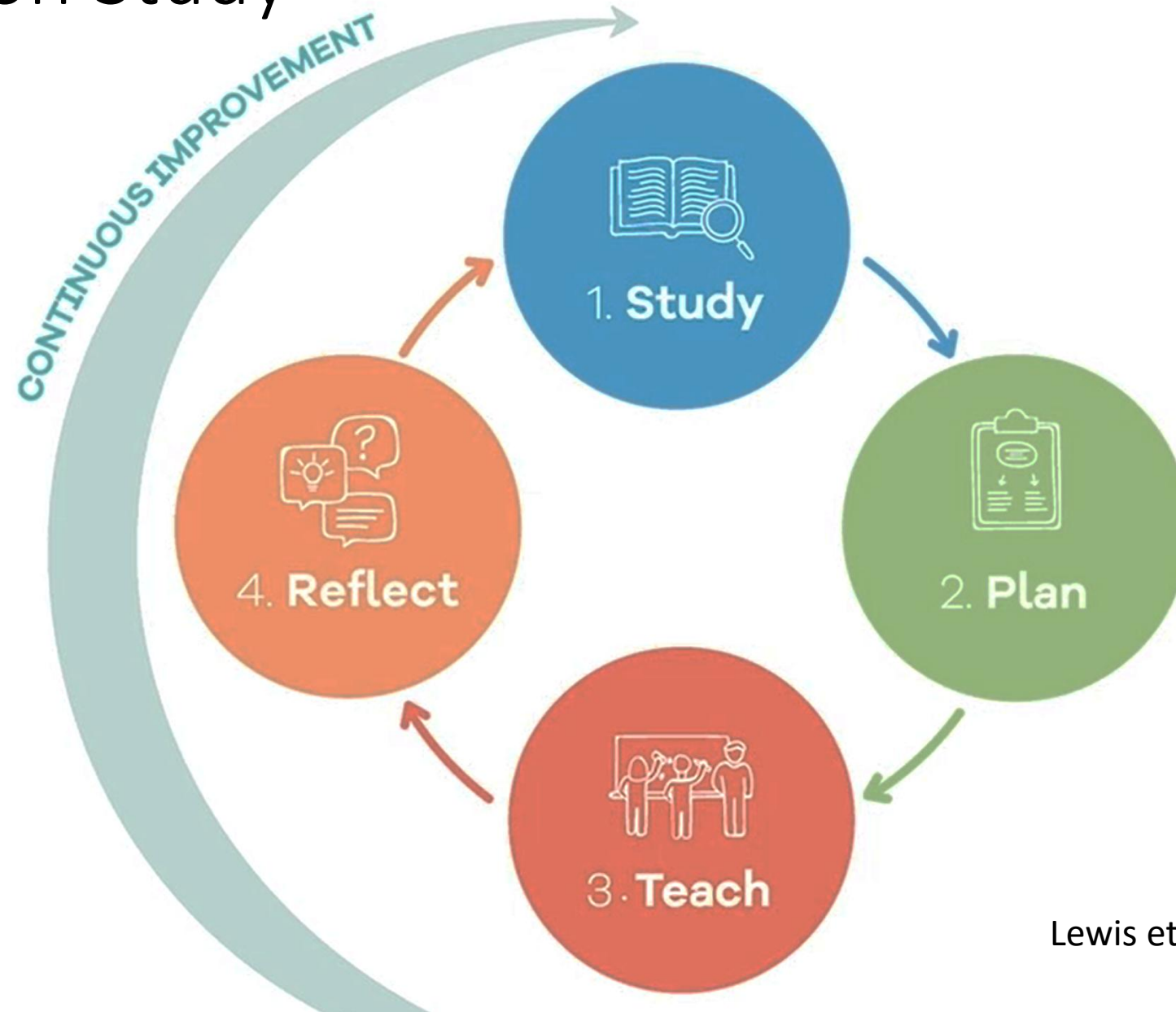
Laboratory of Mathematics

Lesson Study

Teachers collaborating for task design



What is Lesson Study

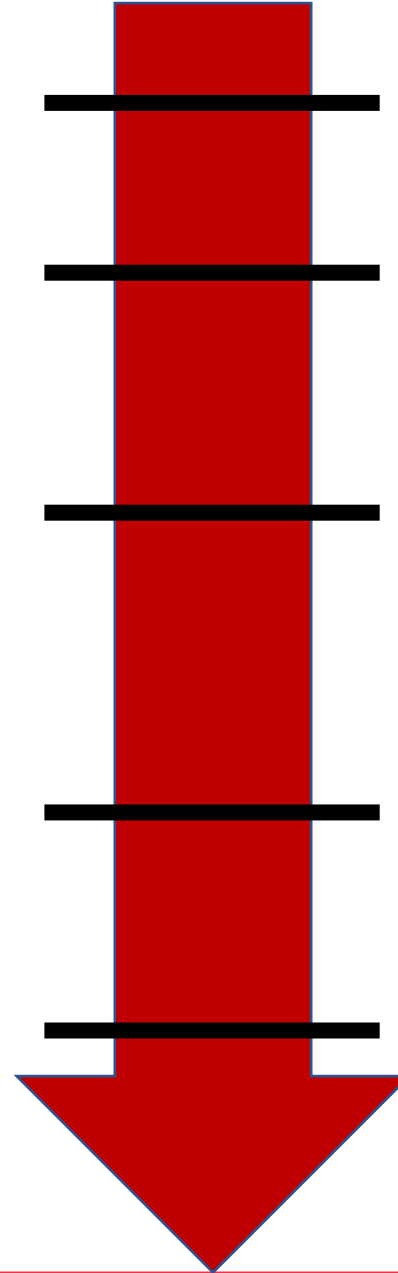
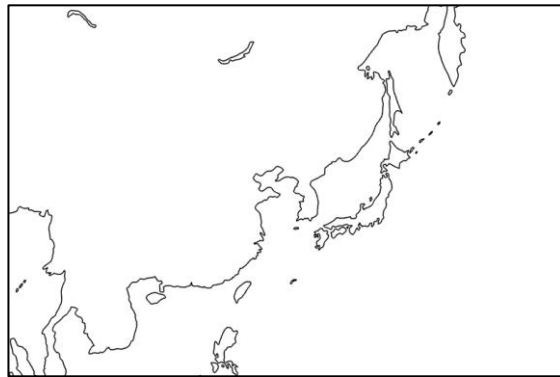


Lewis et al. (2019)

Phases of Lesson Study

1. **Study**: Students' learning goals are decided, and a *research topic* is set to be investigated in relation to such goals. Relevant teaching materials are analysed;
2. **Plan**: A detailed Lesson Plan for a *research lesson* is designed;
3. **Teach**: The research lesson is taught in a real classroom, and observed by other teachers;
4. **Reflect**: The research lesson is discussed, based on data collected during the observation, especially on students' learning.

A brief history of Lesson Study



1872 Emperor Mutsuhito's **reform** of the schooling system

1872 - 1880 Opening of teachers' schools

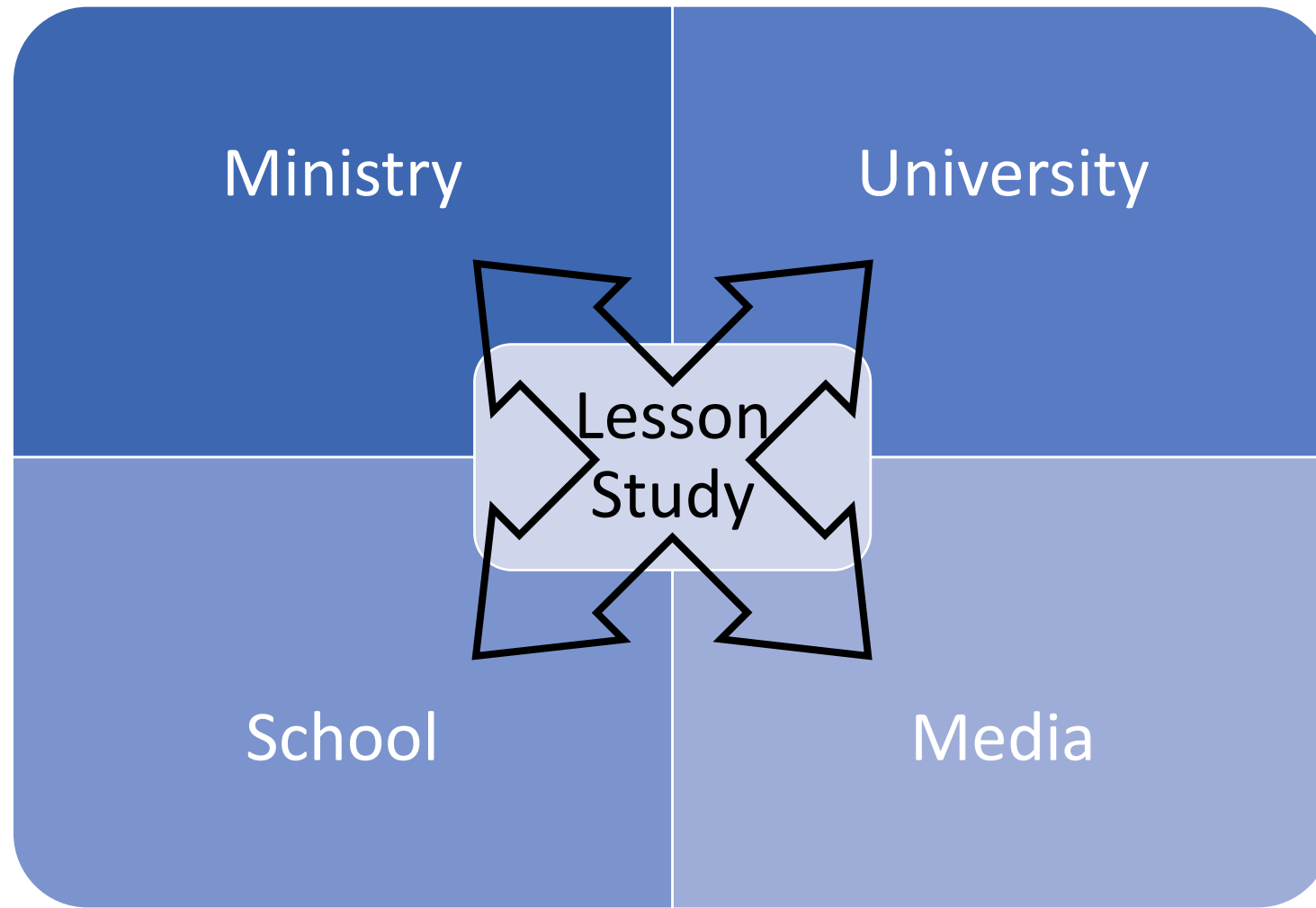
1880 A book on **Pestalozzi's peer-education methods** inspires the first spontaneous Lesson Studies

1900 LS *migrates* to China and Korea

1999 LS meets Western researchers (starting from the USA)

Isoda et al. (2007)

Infrastructure around Lesson Study



Miyakawa and Winsløw (2019)

Mathematics teachers as a nation-wide community



The “lobby” of Mathematics teachers





Lesson Study

One example of task *from* collaboration

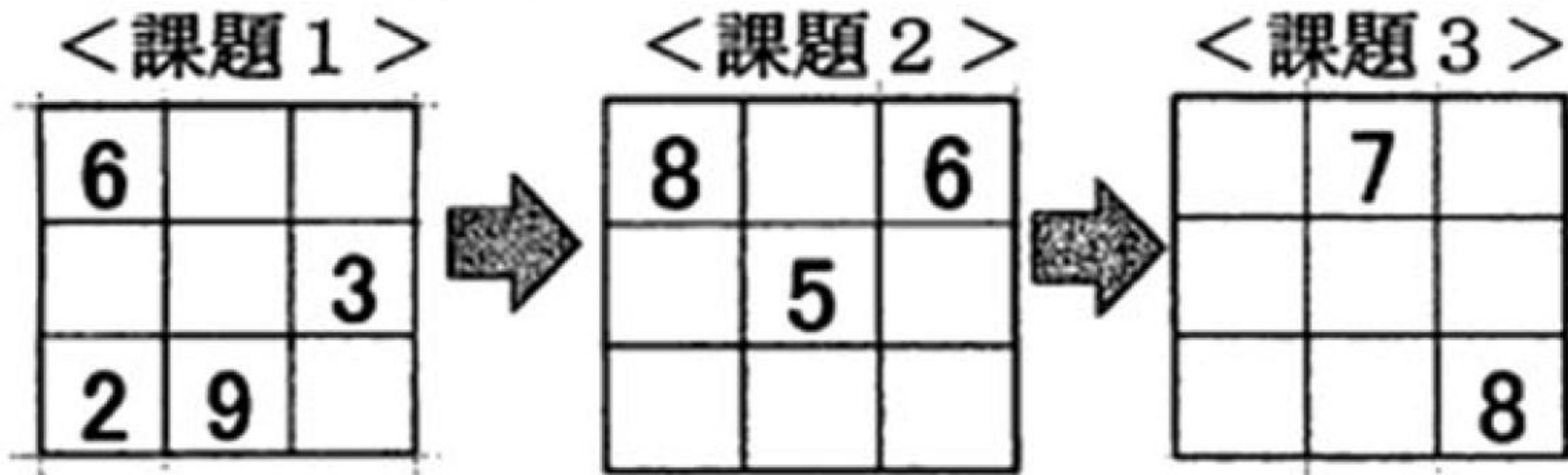
Miyakawa and Winsløw (2019)

Introducing linear equations

- Research problem: *introducing linear equations* through problem solving
- Teaching idea: progressively introduce the need for mathematical modelling
- Methodology: open problem solving
- Grade 7

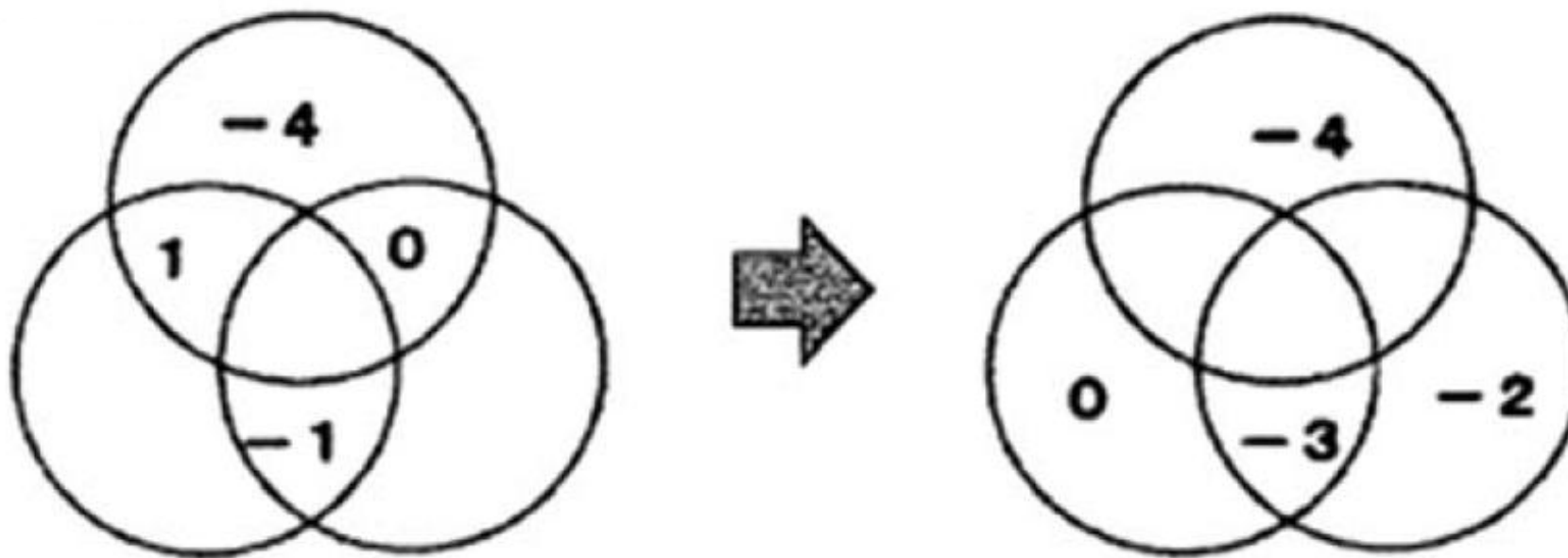
Magic square

- Progressive assignment:
the magic number for each magic square is **15**



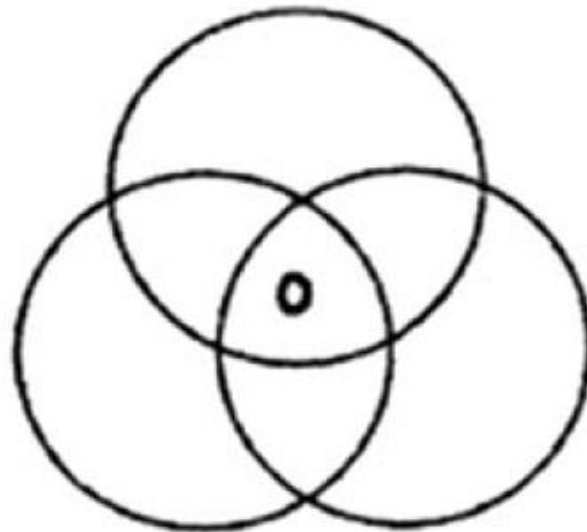
Magic circle\1

- Fill the empty space so that the sum of the numbers in each circle is the same



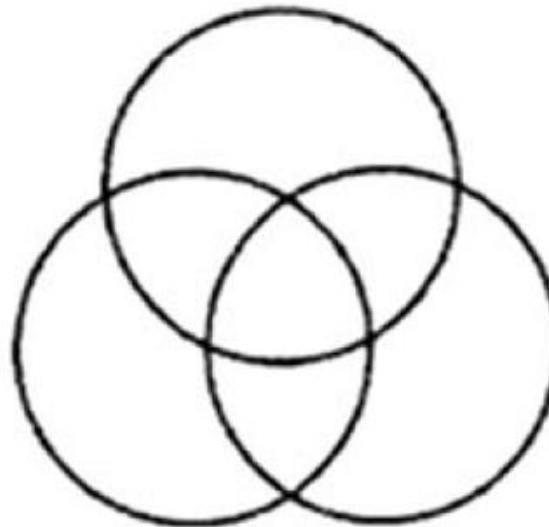
Magic circle\2

- Complete the following magic circle



Magic circle\3

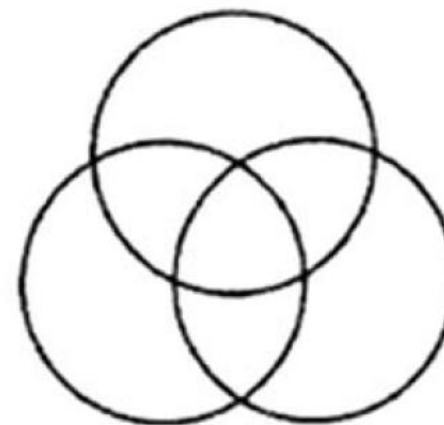
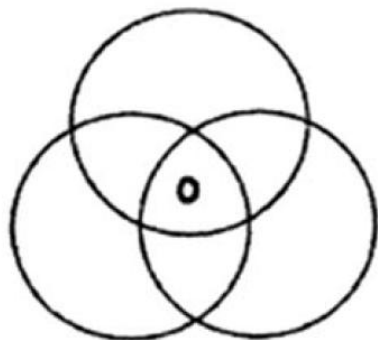
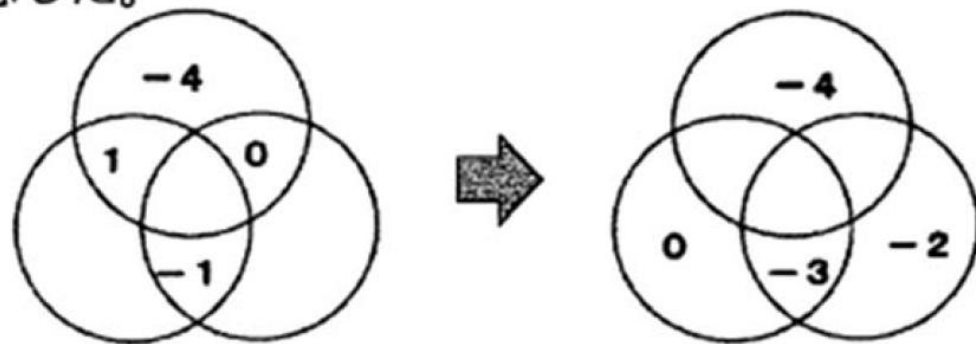
- Create your own set of magic circles and share it with the class as a problem to solve



Pause for thought

What Mathematics are we working with?

7/10/20



Introducing linear equations (???)

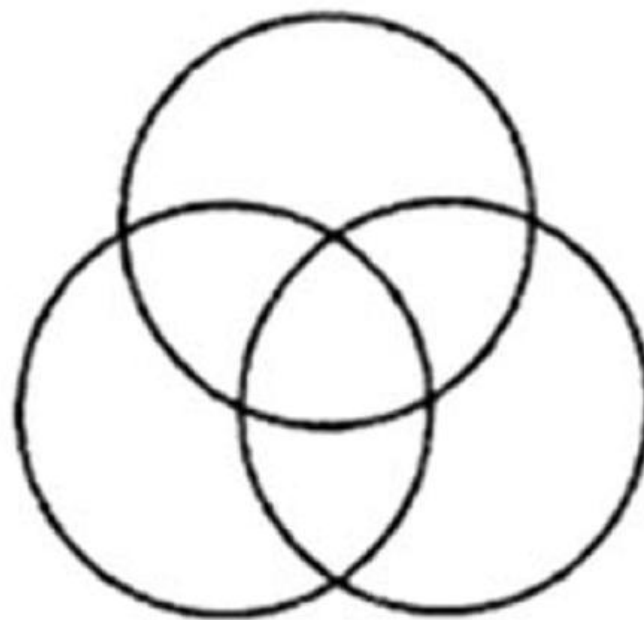
- Unclear connection between the tasks and the topic
- At the introductory stage, students will not see the merit of using equations



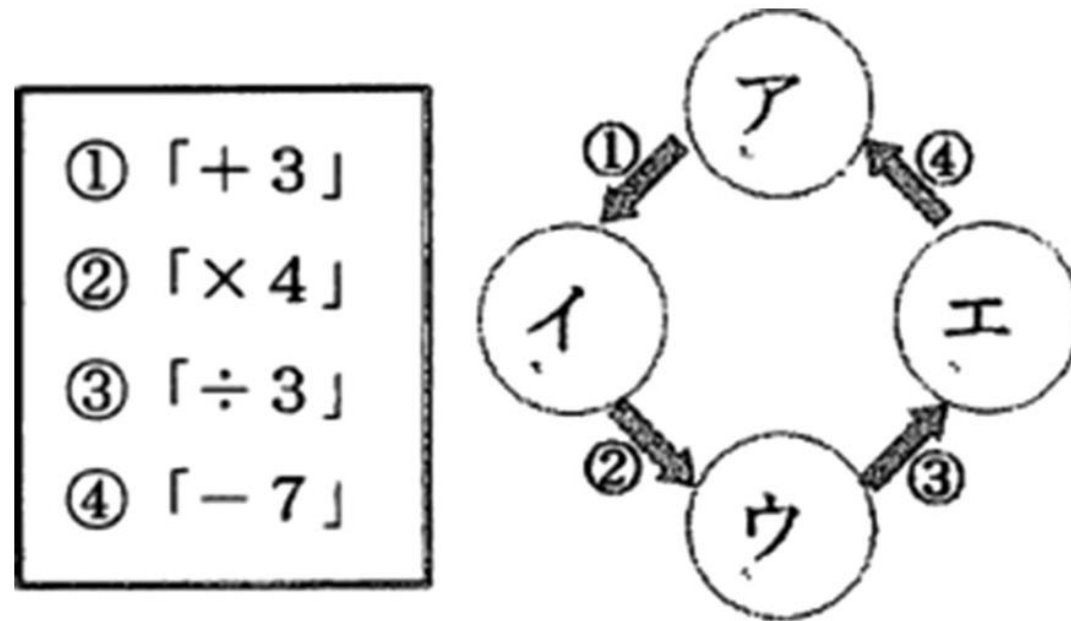
Not really an “introduction”

- The role of the central number is not intuitive
- The model involves simultaneous equations

$$\begin{cases} a + f = b + e \\ c + f = b + d \end{cases}$$

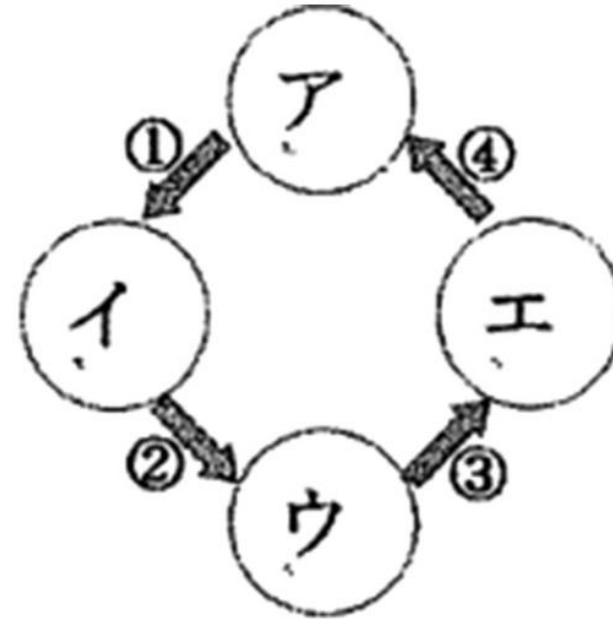
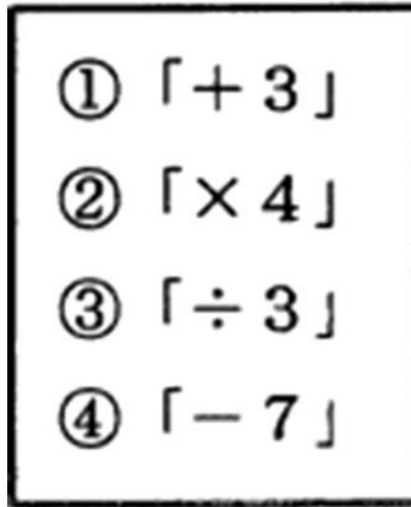


Another idea: calculation square

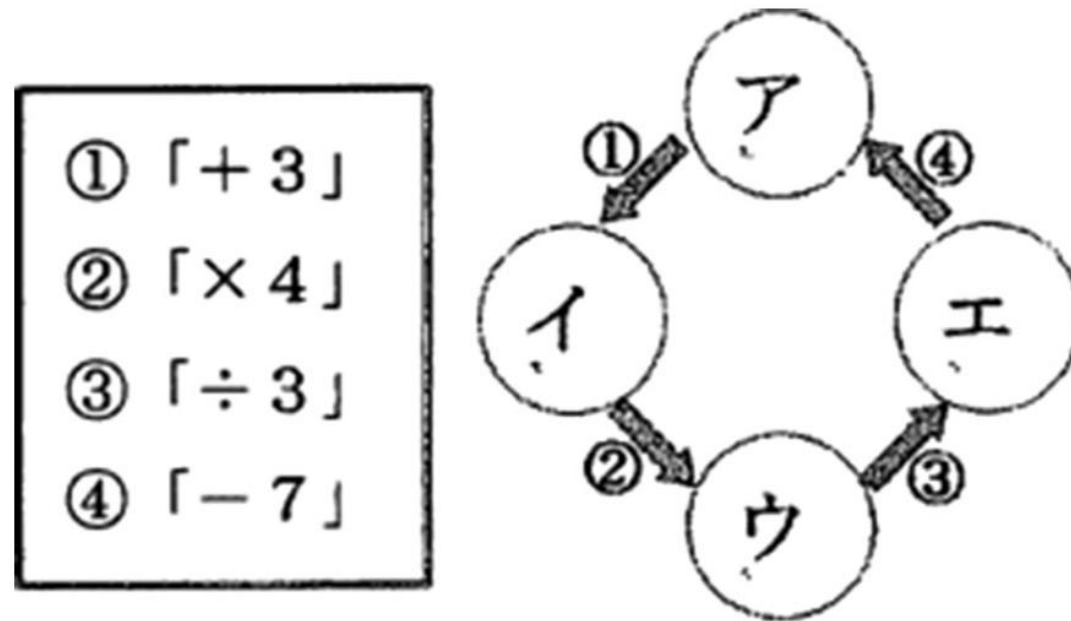


Pause for thought

Why would this work better?



Another idea: calculation square



From trial & error to $\frac{4(x+3)}{3} - 7 = x$



Scuole Secondarie Potenziate in Matematica

Secondary Schools with Enhanced Mathematics Programmes
Teachers collaborating for task analysis



National and local project



Image source: SuperMap.World



Image source: Wikimedia

The national



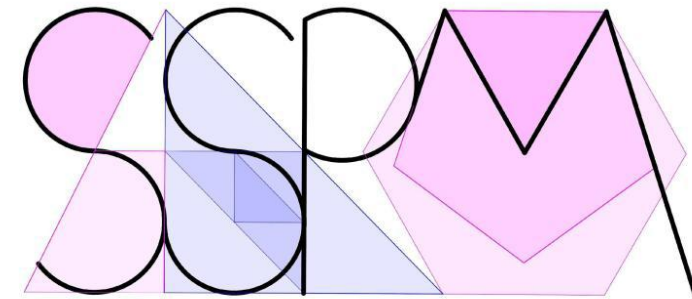
Image source: SuperMap.World

- 27 universities
- Italian Mathematics Union
- Network of schools
- >150 high-schools
- National congress every year
- Multiple events of public engagement at the national level

The local



Image source: Wikimedia



- >2000 pupils
- >100 teachers
- >50 schools (primary, middle and high schools)
- For the students: 33 additional hours of laboratory of mathematics
- For the teachers: 20 hours of professional development +
- Database with 110 activities +
- Introduction to research in Mathematics Education +
- Career growth as teacher-researchers

The local group



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DI MATEMATICA
GIUSEPPE PEANO
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Scuole Secondarie Potenziate in Matematica

Secondary Schools with Enhanced Mathematics Programmes

Two examples of tasks *for* collaboration

Hands-on: task one

Introduction to the laboratory of mathematics

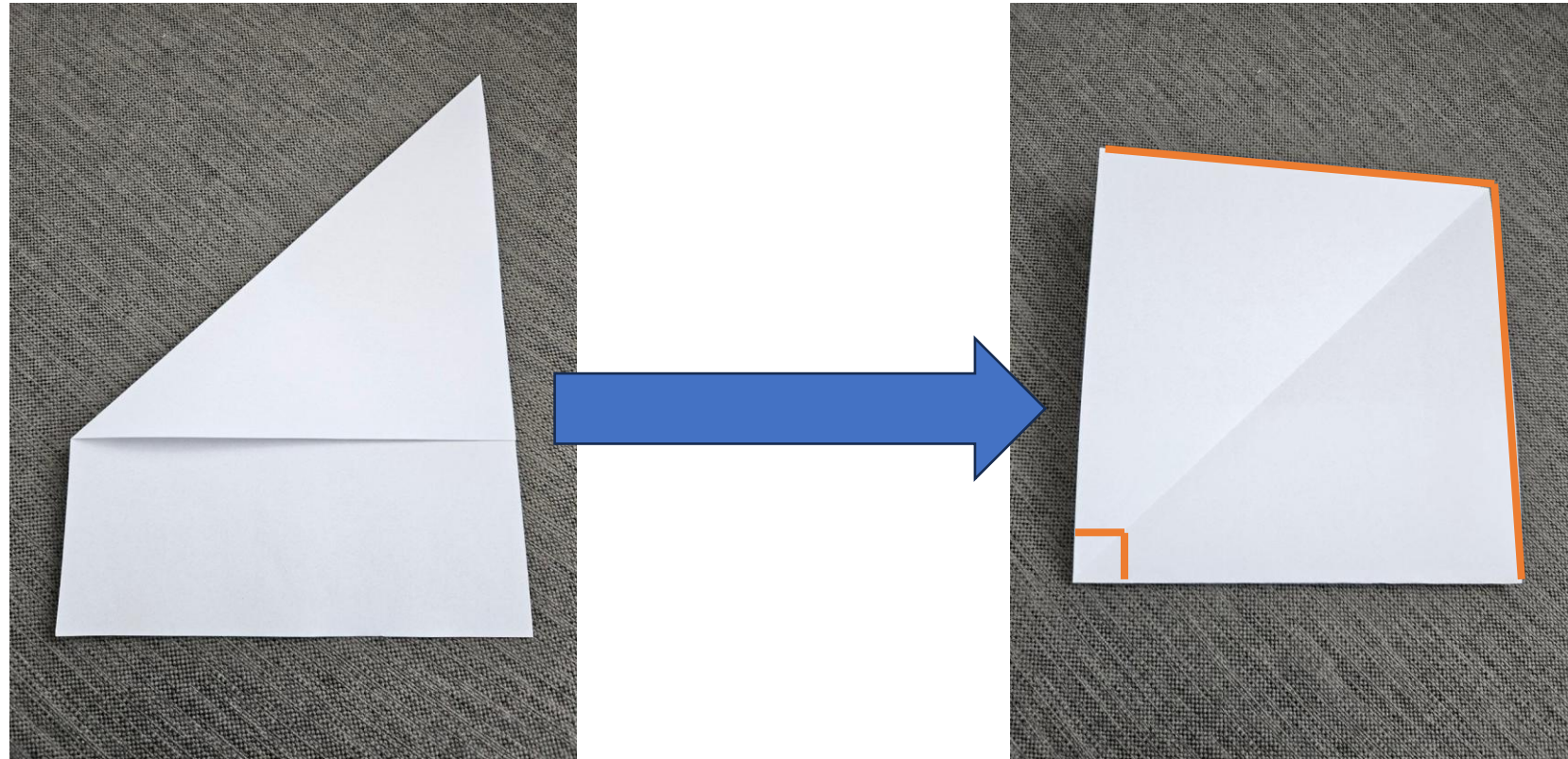
Tri-angular discussion

Step 1

Take the blank sheet.

Cut from it the biggest square possible. How do we do it?

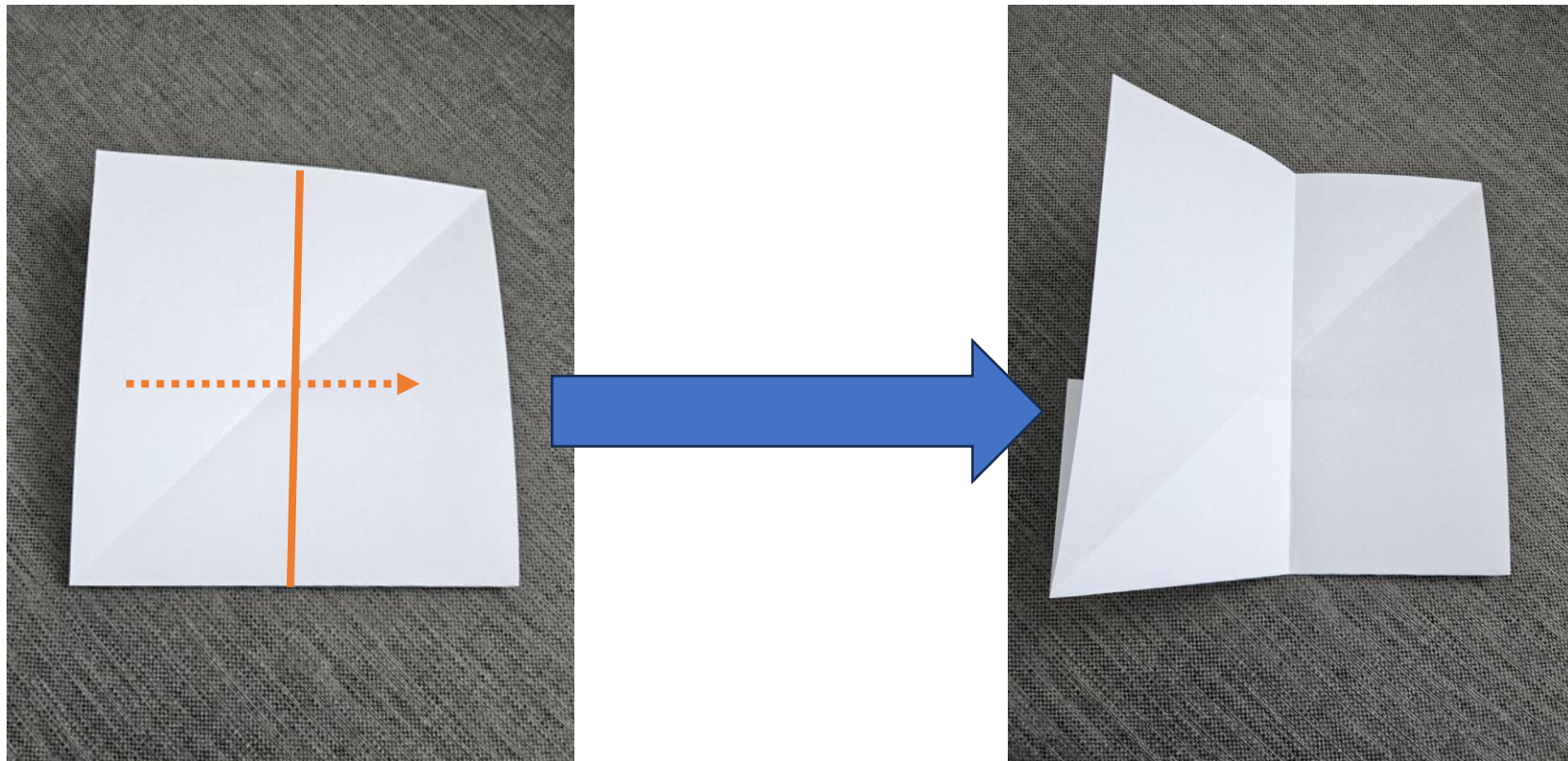
Step 1



This is a square because these two sides overlap (therefore have the same length) and the angles are right angles by construction.

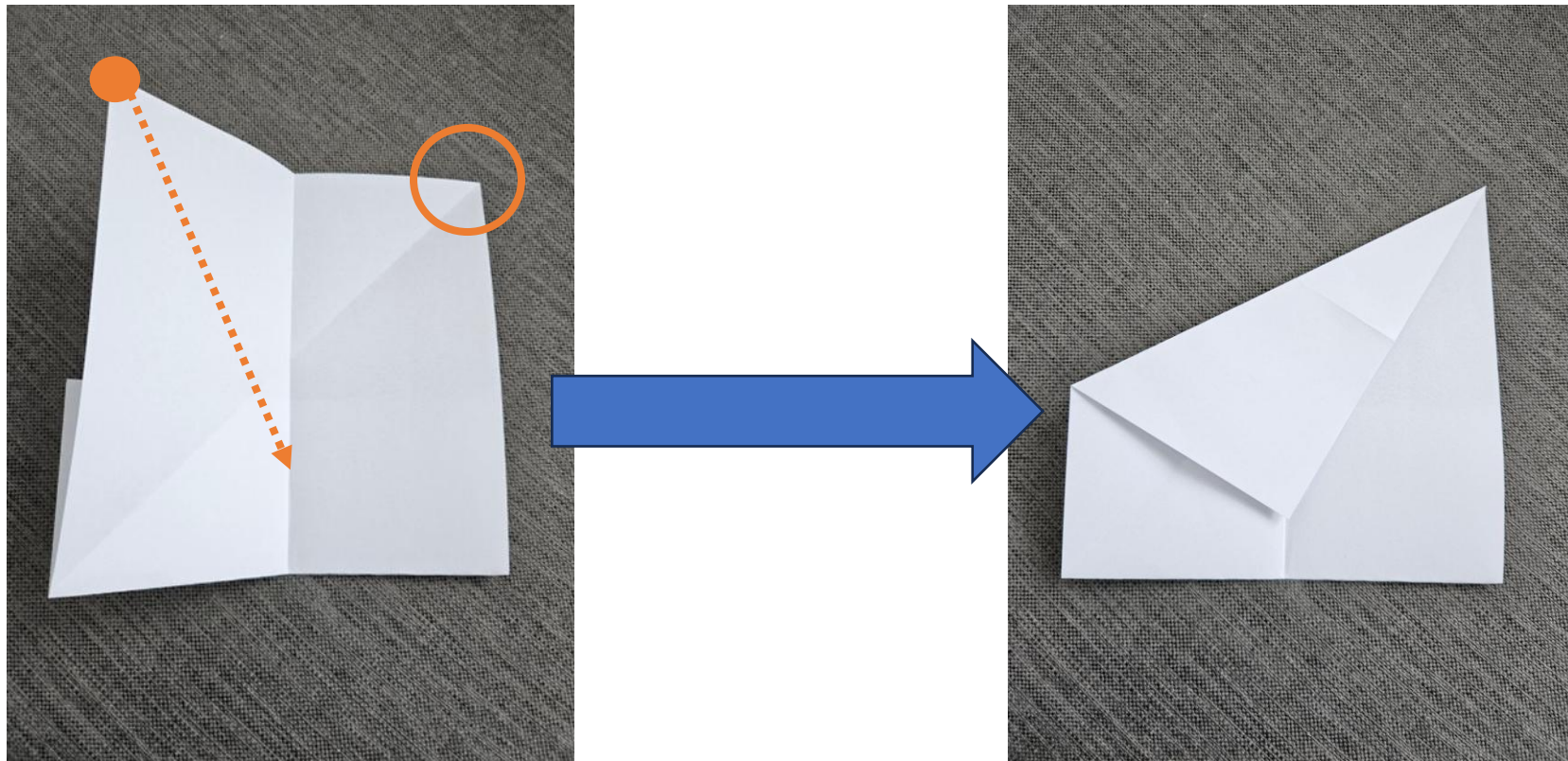
Step 2

Fold the square in half, on the mid-point of two opposite sides.



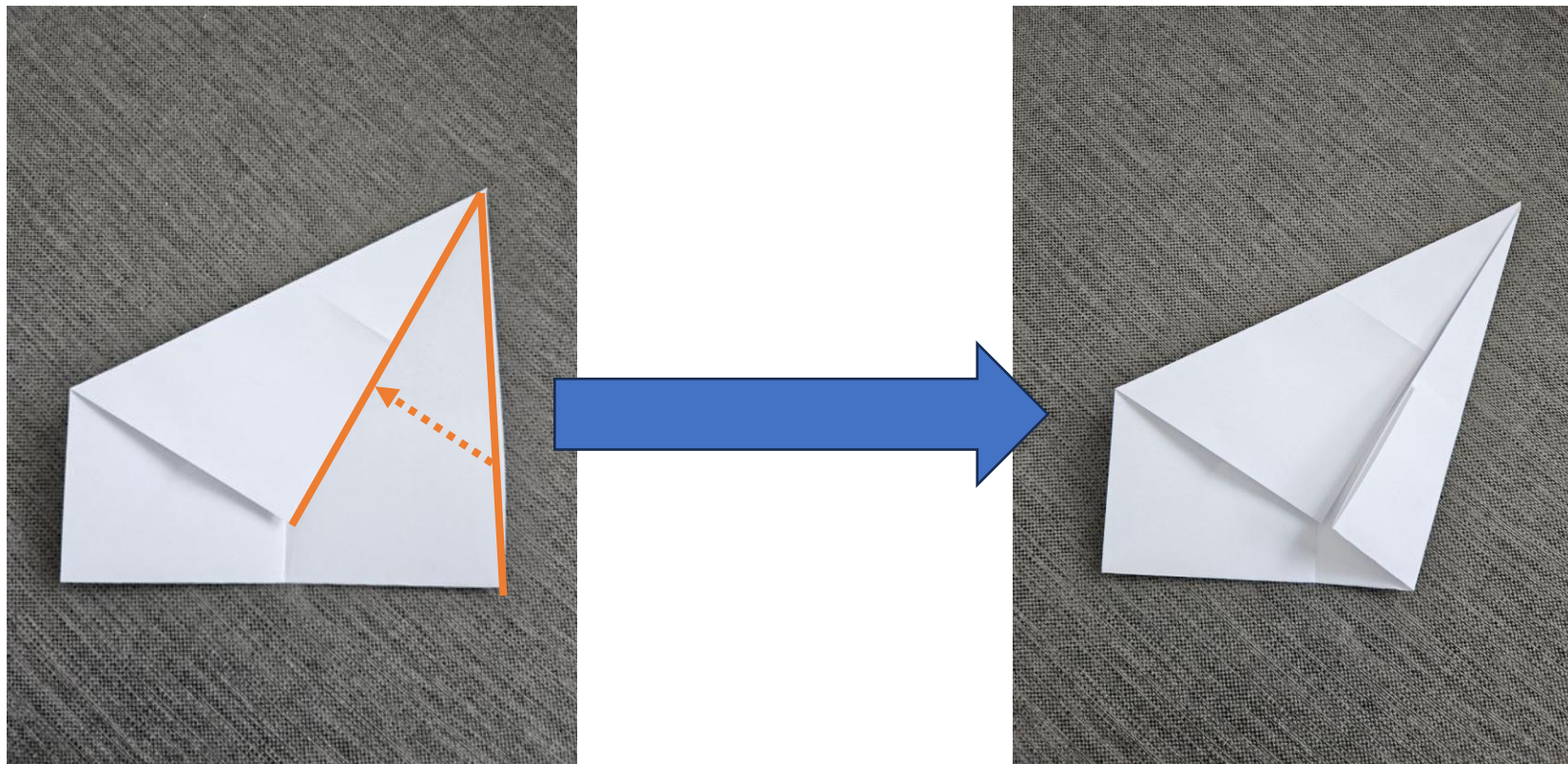
Step 3

Cut this angle by folding this vertex on the previous folding line.



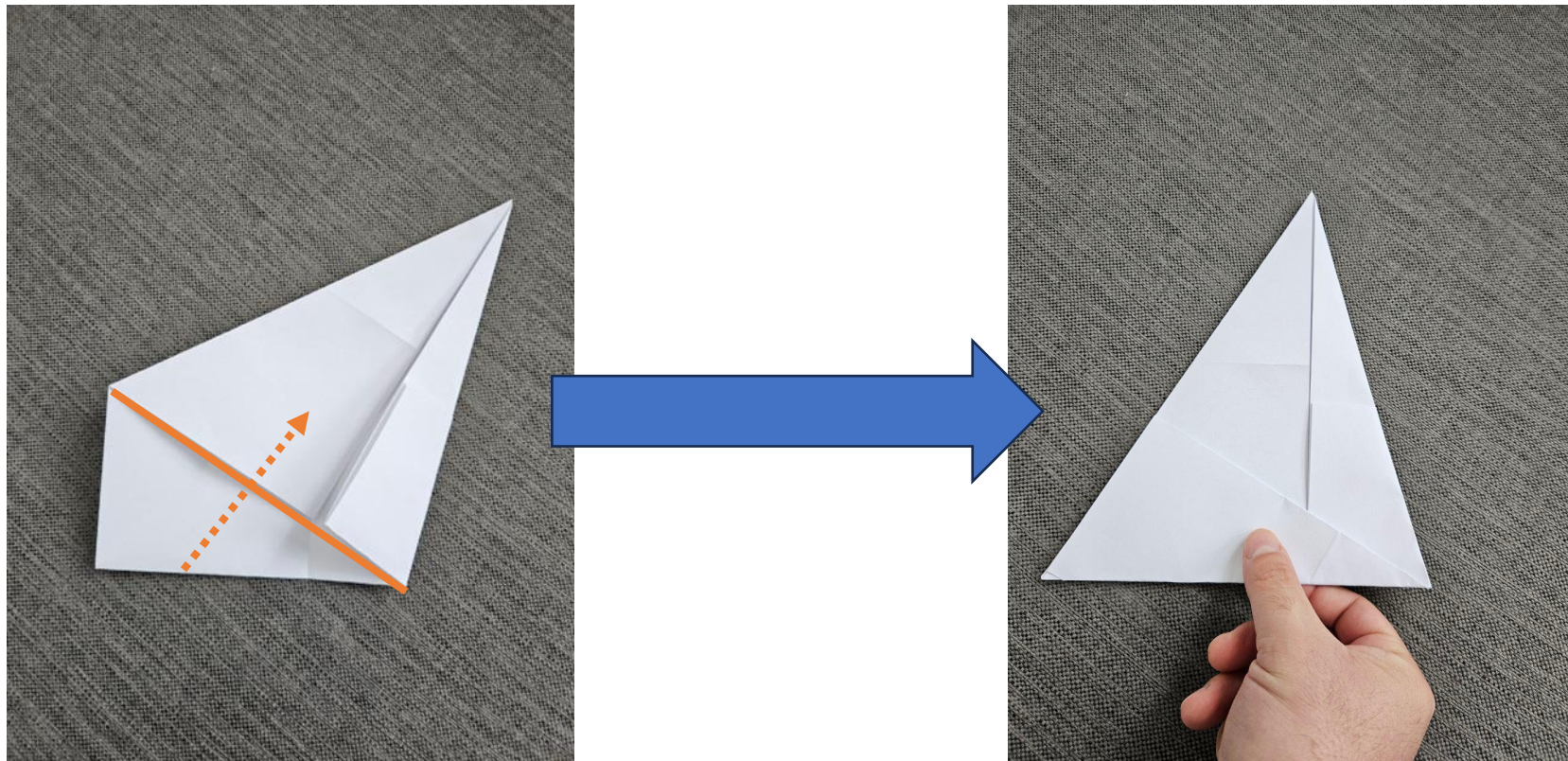
Step 4

Overlap these two sides.

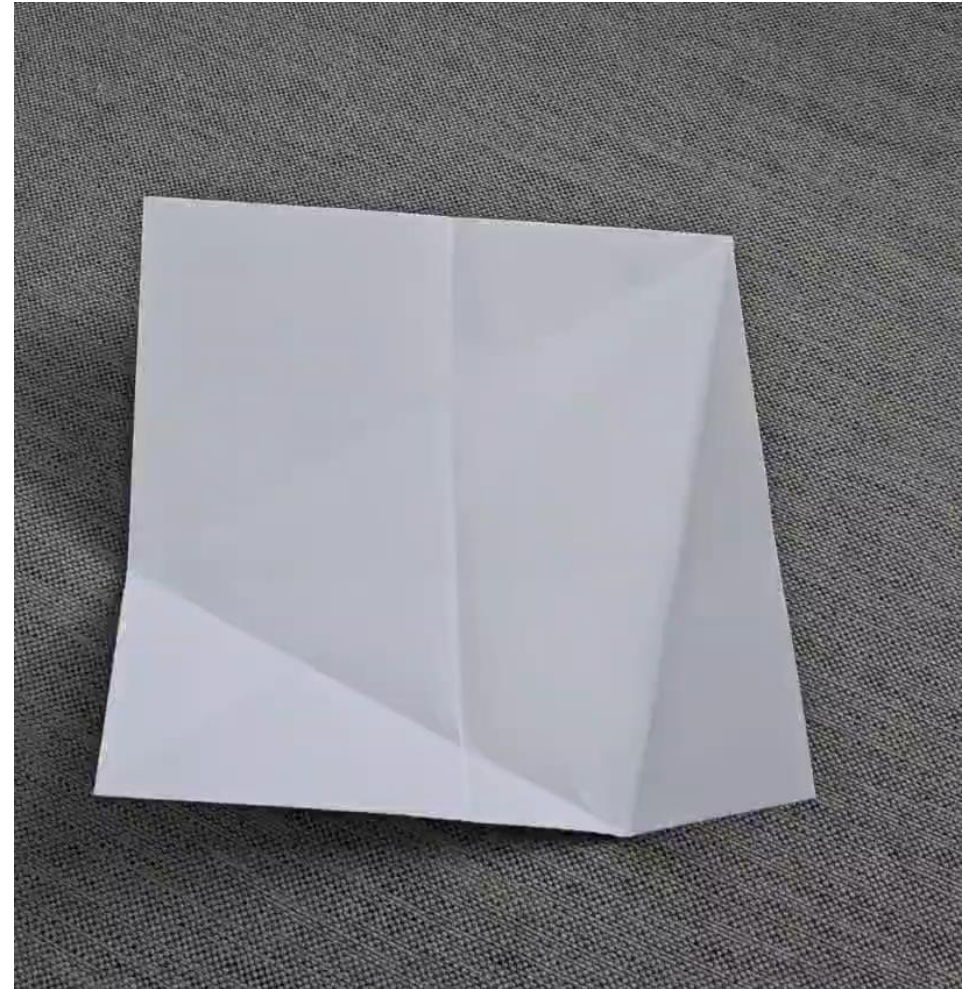


Step 5

Finally, fold over this side.



Pit-stop for a check, in case you need some help



Task 2



Find the measure of each angle of this triangle. Justify your answers by adequately explaining all the steps you went through. You can use the geometrical properties of the folding that we did.

No protractors and no eyeballing.

You need to be able to explain your reasoning to the other groups.

10' group work

Elect a representative to present your answer

You can discuss in Polish

Students: group presentations

We listen to the presentation of each group.

We take notes of steps that are correct, but different from the ones we followed.

We take notes of steps that do not convince us, and think why they do not convince us.

We discuss our observations.

What is the point?

Developing social skills:

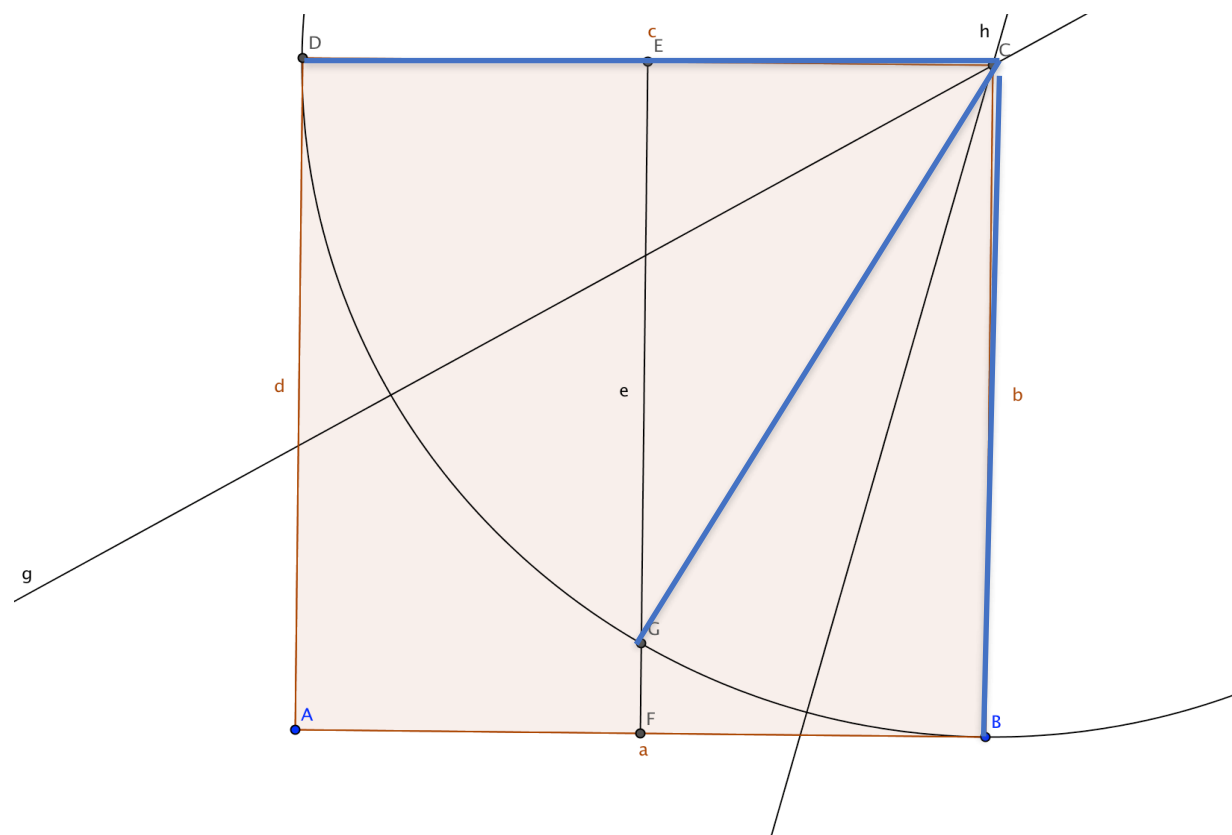
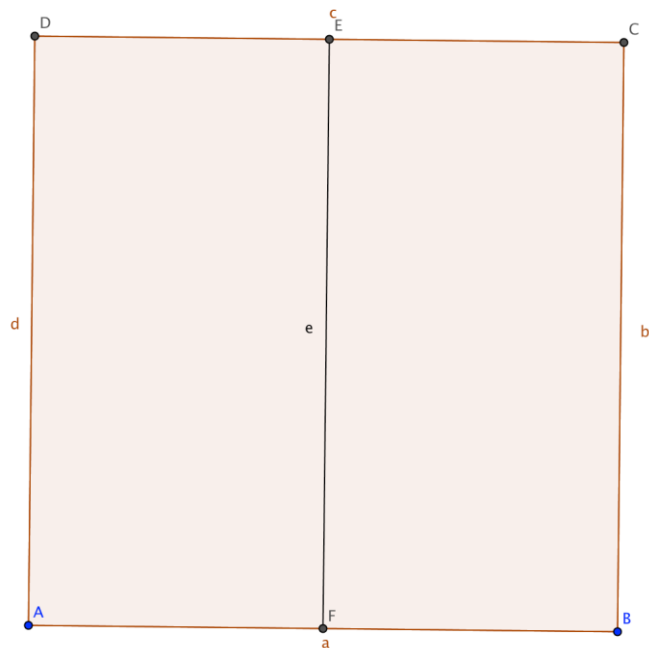
- listening
- discussion
- argumentation

Developing mathematical skills:

- mathematical modelling
- abstraction
- proof

Evolving knowledge: previous knowledge is reapplied in an unknown situation to develop problem solving

Further development: GeoGebra



Analysis and discussion

Task two

Introduction to task analysis

Towards the integral

Your task

- 10 minutes to read the material on your own: up to task 1. If you have time, also read task 2.
 - Consider its application in the classroom: when to do it, why, how it connects to the curriculum, what students need to know to approach the tasks...
- 10 minutes turn to your neighbour to share your observations
- 10 minutes sharing with the whole group

What is the point? Teachers' edition

Developing teaching skills:
new methodologies
new approaches
new teaching ideas

Developing meta-teaching skills: developing the ability to
analyse tasks with a critical standpoint

Before we finish, a question for you



<https://forms.gle/aoSdXPobEsTPzaUb9>

References

Giacardi, L. (2011). L'emergere dell'idea di laboratorio di matematica agli inizi del Novecento. *Il Curriculum Di Matematica e Fisica Nella Scuola Del III Millennio: Infanzia, Primaria, Secondaria Di Primo e Di Secondo Livello*, 55–65.

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Miyakawa, T., & Winsløw, C. (2019). Paradidactic infrastructure for sharing and documenting mathematics teacher knowledge: A case study of “practice research” in Japan. *Journal of Mathematics Teacher Education*, 22(3), 281–303. <https://doi.org/10.1007/s10857-017-9394-y>

Website of Liceo Matematico - <https://www.liceomatematico.it/>

Website of Scuole Secondarie Potenziate in Matematica - <https://difima.i-learn.unito.it/course/view.php?id=159>



Thank you very much!

Grazie mille!

Dziękuję bardzo!

ありがとうございました

