

Twierdzenie Karola Omyłka

Sielpia Wielka

21 X 2023

Każda liczba, która kończy się na n ,
jest podzielna przez n .

Karol Omytek

1

2

5

~~0~~

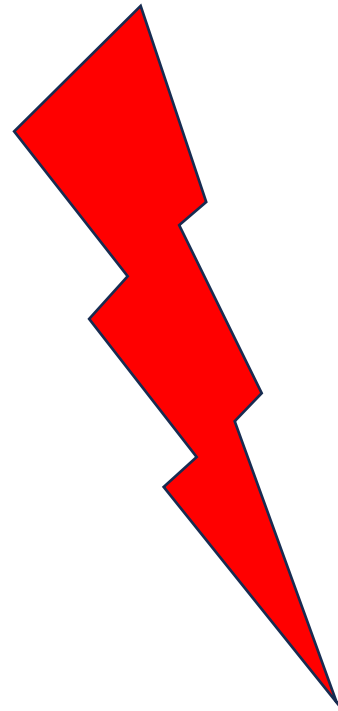
10

20, 50, 100

3

4

...



1, 2, 5, 10, 20, 50, 100,

200, 500, 1000,

$c \cdot 10^i$, gdzie $c \in \{1, 2, 5\}$, $i \in \mathbb{N}$

25, 250, 2500,

$c \cdot 10^i$, gdzie $c \in \{1, 2, 5, 5^2\}$

125, 1250, 12500,

$c \cdot 10^i$, gdzie $c \in \{1, 2, 5, 5^2, 5^3\}$

4, 8, 40, 80

$c \cdot 10^i$, gdzie $c \in \{1, 2, 2^2, 2^3, 5, 5^2, 5^3\}$

625, ...

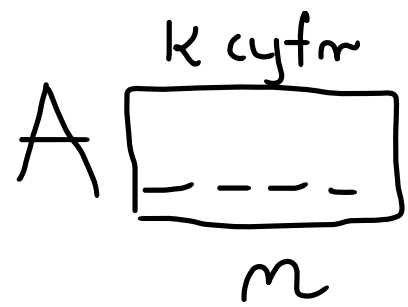
1625

$c \cdot 10^i$, gdzie $c \in \{1, 2, 5, 5^2, 5^3, 5^4, \dots\}$

???

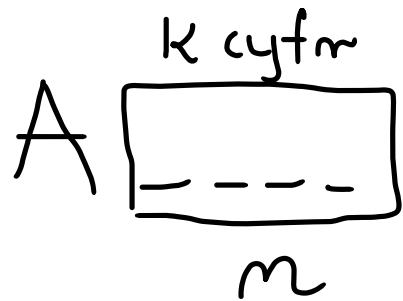
Tylko liczby, które kończą się na $c \cdot 10^i$, gdzie $c \in \{1, 2, 5, 5^2, 5^3\}$, są podzielne przez $c \cdot 10^i$.

Karol Omyłek



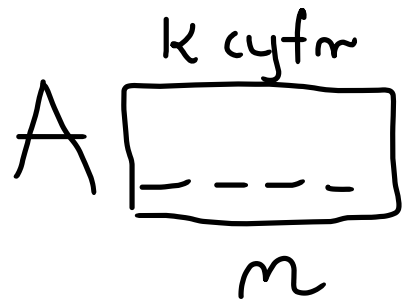
A $\underbrace{\hspace{2cm}}_{m}$ k cyfr

A $\underbrace{00\dots00}_k + n$



$$A \underbrace{00\dots00}_k + m$$

$$= A \cdot 10^k + \underbrace{m}_{k \text{ cyfr}}$$

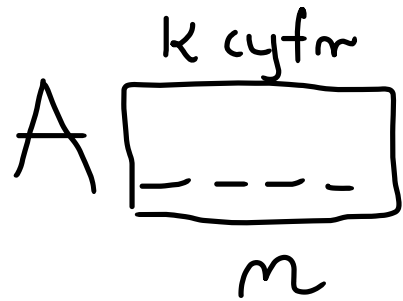


$$A \underbrace{00\dots00}_k + m$$

$$= A \cdot 10^k + \underbrace{m}_{k \text{ cyfr}}$$

$$m / 10^k$$

i n ma k cyfr



$$A \underbrace{00\dots00}_k + n$$

$$= A \cdot 10^k + \underbrace{n}_{k \text{ cyfr}}$$

$$n \mid 10^k$$

i n ma k cyfr

$$n = 2^a \cdot 5^b$$

$$\mathcal{K} = \{k\text{-cyfrowe okienki } 10^k\}$$

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$$= \{m \in \mathbb{N} : 10^{k-1} \leq m\}$$

$$\mathcal{K} = \{k\text{-cyfrowe okienki } 10^k\}$$
$$= \left\{ m \in \mathbb{N} : 10^{k-1} \leq m \leq \frac{10^k}{2} \right\}$$

$$\mathcal{K} = \{k\text{-cyfrowe dzielniki } 10^k\}$$

$$= \left\{ m \in \mathbb{N} : 10^{k-1} \leq m \leq \frac{10^k}{2} \wedge m = 2^a \cdot 5^b \right\}$$

$$\mathcal{K} = \{k\text{-cyfrowe dzielniki } 10^k\}$$

$$= \left\{ m \in \mathbb{N} : 10^{k-1} \leq m \leq \frac{10^k}{2} \wedge m = 2^a \cdot 5^b \right\}$$

$$= \left\{ m \in \mathbb{N} : 10^{k-1} \leq m \leq 5 \cdot 10^{k-1} \wedge m = 2^a \cdot 5^b \right\}$$

$$i) a = b \Rightarrow a = k - 1$$

$$i) a = b \Rightarrow a = k-1$$

$$10^{k-1} \leq 2 \cdot 5^{k-1} \leq 5 \cdot 10^{k-1}$$

a olowolme



$$i) a = b \Rightarrow a = k - 1$$

$$10^{k-1} \leq 2 \cdot 5^{k-1} \leq 5 \cdot 10^{k-1} \quad \square$$

$$ii) a > b \Rightarrow a = k$$

$$i) a = b \Rightarrow a = k - 1$$

$$10^{k-1} \leq 2^{k-1} \cdot 5^{k-1} \leq 5 \cdot 10^{k-1} \quad \square$$

$$ii) a > b \Rightarrow a = k$$

$$10^{k-1} \leq 2^k \cdot 5^b \leq 5 \cdot 10^{k-1}$$

$$i) a = b \Rightarrow a = k - 1$$

$$10^{k-1} \leq 2^{k-1} \cdot 5^{k-1} \leq 5 \cdot 10^{k-1} \quad \square$$

$$ii) a > b \Rightarrow a = k$$

$$10^{k-1} \leq 2^k \cdot 5^b \leq 5 \cdot 10^{k-1}$$

$$10^{a-1} \leq 2^a \cdot 5^b \leq 5 \cdot 10^{a-1} \quad /: 10^b$$

$$i) a = b \Rightarrow a = k - 1$$

$$10^{k-1} \leq 2^{k-1} \cdot 5^{k-1} \leq 5 \cdot 10^{k-1} \quad \square$$

$$ii) a > b \Rightarrow a = k$$

$$10^{k-1} \leq 2^k \cdot 5^b \leq 5 \cdot 10^{k-1}$$

$$10^{a-1} \leq 2^a \cdot 5^b \leq 5 \cdot 10^{a-1} \quad /: 10^b$$

$$10^{a-b-1} \leq 2^{a-b} \leq 5 \cdot 10^{a-b-1} \quad /: 2^{a-b-1}$$

$$i) a = b \Rightarrow a = k - 1$$

$$10^{k-1} \leq 2^{k-1} \cdot 5^{k-1} \leq 5 \cdot 10^{k-1} \quad \square$$

$$ii) a > b \Rightarrow a = k$$

$$10^{k-1} \leq 2^k \cdot 5^b \leq 5 \cdot 10^{k-1}$$

$$10^{a-1} \leq 2^a \cdot 5^b \leq 5 \cdot 10^{a-1} \quad /: 10^b$$

$$10^{a-b-1} \leq 2^{a-b} \leq 5 \cdot 10^{a-b-1} \quad /: 2^{a-b-1}$$

$$5^{a-b-1} \leq 2 \leq 5^{a-b}$$

$$i) a = b \Rightarrow a = k - 1$$

$$10^{k-1} \leq 2^{k-1} \cdot 5^{k-1} \leq 5 \cdot 10^{k-1} \quad \square$$

$$ii) a > b \Rightarrow a = k$$

$$10^{k-1} \leq 2^k \cdot 5^b \leq 5 \cdot 10^{k-1}$$

$$10^{a-1} \leq 2^a \cdot 5^b \leq 5 \cdot 10^{a-1} \quad /: 10^b$$

$$10^{a-b-1} \leq 2^{a-b} \leq 5 \cdot 10^{a-b-1} \quad /: 2^{a-b-1}$$

$$5^{\underbrace{a-b-1}_{\neq 0}} \leq 2 \leq 5^{\underbrace{a-b}_{\neq 1}}$$

$$i) a = b \Rightarrow a = k - 1$$

$$10^{k-1} \leq 2^{k-1} \cdot 5^{k-1} \leq 5 \cdot 10^{k-1} \quad \square$$

$$ii) a > b \Rightarrow a = k$$

$$10^{k-1} \leq 2^k \cdot 5^b \leq 5 \cdot 10^{k-1}$$

$$10^{a-1} \leq 2^a \cdot 5^b \leq 5 \cdot 10^{a-1} \quad /: 10^b$$

$$10^{a-b-1} \leq 2^{a-b} \leq 5 \cdot 10^{a-b-1} \quad /: 2^{a-b-1}$$

$$5^{\underbrace{a-b-1}_0} \leq 2 \leq 5^{\underbrace{a-b}_1}$$

$$a-b \leq 1 \quad \wedge \quad a-b \geq 1$$

$$i) a = b \Rightarrow a = k - 1$$

$$10^{k-1} \leq 2^{k-1} \cdot 5^{k-1} \leq 5 \cdot 10^{k-1} \quad \square$$

$$ii) a > b \Rightarrow a = k$$

$$10^{k-1} \leq 2^k \cdot 5^b \leq 5 \cdot 10^{k-1}$$

$$10^{a-1} \leq 2^a \cdot 5^b \leq 5 \cdot 10^{a-1} \quad /: 10^b$$

$$10^{a-b-1} \leq 2^{a-b} \leq 5 \cdot 10^{a-b-1} \quad /: 2^{a-b-1}$$

$$5^{\underbrace{a-b-1}_0} \leq 2 \leq 5^{\underbrace{a-b}_1}$$

$$a-b \leq 1 \quad \wedge$$

$$a-b \geq 1$$

$$\underline{a-b=1} \quad \square$$

$$\text{iii) } a < b \implies b = k$$

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$$10^{k-1} \leq 2^a \cdot 5^k \leq 5 \cdot 10^{k-1}$$

$$\text{iii) } a < b \implies b = k$$

$$10^{k-1} \leq 2^a \cdot 5^k \leq 5 \cdot 10^{k-1}$$

$$10^{b-1} \leq 2^a \cdot 5^b \leq 5 \cdot 10^{b-1} \quad /: 10^a$$

$$\text{iii) } a < b \implies b = k$$

$$10^{k-1} \leq 2^a \cdot 5^k \leq 5 \cdot 10^{k-1}$$

$$10^{b-1} \leq 2^a \cdot 5^b \leq 5 \cdot 10^{b-1} \quad /: 10^a$$

$$10^{b-a-1} \leq 5^{b-a} \leq 5 \cdot 10^{b-a-1}$$

$$\text{iii) } a < b \Rightarrow b = k$$

$$10^{k-1} \leq 2^a \cdot 5^k \leq 5 \cdot 10^{k-1}$$

$$10^{b-1} \leq 2^a \cdot 5^b \leq 5 \cdot 10^{b-1} \quad /: 10^a$$

$$10^{b-a-1} \leq 5^{b-a} \leq 5 \cdot 10^{b-a-1}$$

$$10^{b-a-1} \leq 5^{b-a} \quad /: 5^{b-a-1} \quad \wedge \quad 5^{b-a} \leq 5 \cdot 10^{b-a-1} \quad /: 5^{b-a}$$

$$\text{iii) } a < b \Rightarrow b = k$$

$$10^{k-1} \leq 2^a \cdot 5^k \leq 5 \cdot 10^{k-1}$$

$$10^{b-1} \leq 2^a \cdot 5^b \leq 5 \cdot 10^{b-1} \quad /: 10^a$$

$$10^{b-a-1} \leq 5^{b-a} \leq 5 \cdot 10^{b-a-1}$$

$$10^{b-a-1} \leq 5^{b-a} \quad /: 5^{b-a-1}$$

$$2^{b-a-1} \leq 5$$

\wedge

$$5^{b-a} \leq 5 \cdot 10^{b-a-1} \quad /: 5^{b-a}$$

\wedge

$$1 \leq 2^{b-a-1}$$

$$\text{iii) } a < b \Rightarrow b = k$$

$$10^{k-1} \leq 2^a \cdot 5^k \leq 5 \cdot 10^{k-1}$$

$$10^{b-1} \leq 2^a \cdot 5^b \leq 5 \cdot 10^{b-1} \quad /: 10^a$$

$$10^{b-a-1} \leq 5^{b-a} \leq 5 \cdot 10^{b-a-1}$$

$$10^{b-a-1} \leq 5^{b-a} \quad /: 5^{b-a-1}$$

$$2^{b-a-1} \leq 5$$

\wedge

$$5^{b-a} \leq 5 \cdot 10^{b-a-1} \quad /: 5^{b-a}$$

$$1 \leq 2^{b-a-1}$$

\wedge

$$1 \leq 2^{b-a-1} \leq 5$$

$$\text{iii) } a < b \Rightarrow b = k$$

$$10^{k-1} \leq 2^a \cdot 5^k \leq 5 \cdot 10^{k-1}$$

$$10^{b-1} \leq 2^a \cdot 5^b \leq 5 \cdot 10^{b-1} \quad /: 10^a$$

$$10^{b-a-1} \leq 5^{b-a} \leq 5 \cdot 10^{b-a-1}$$

$$10^{b-a-1} \leq 5^{b-a} \quad /: 5^{b-a-1}$$

$$2^{b-a-1} \leq 5$$

\wedge

$$5^{b-a} \leq 5 \cdot 10^{b-a-1} \quad /: 5^{b-a}$$

$$1 \leq 2^{b-a-1}$$

\wedge

$$1 \leq 2^{b-a-1} \leq 5$$

$$0 \leq b-a-1 \leq 2$$

$$\text{iii) } a < b \Rightarrow b = k$$

$$10^{k-1} \leq 2^a \cdot 5^k \leq 5 \cdot 10^{k-1}$$

$$10^{b-1} \leq 2^a \cdot 5^b \leq 5 \cdot 10^{b-1} \quad /: 10^a$$

$$10^{b-a-1} \leq 5^{b-a} \leq 5 \cdot 10^{b-a-1}$$

$$10^{b-a-1} \leq 5^{b-a} \quad /: 5^{b-a-1}$$

$$2^{b-a-1} \leq 5$$

\wedge

$$5^{b-a} \leq 5 \cdot 10^{b-a-1} \quad /: 5^{b-a}$$

$$1 \leq 2^{b-a-1}$$

\wedge

$$1 \leq 2^{b-a-1} \leq 5$$

$$0 \leq b-a-1 \leq 2$$

$$1 \leq b-a \leq 3 \quad \square$$

$$\mathcal{K} = \{k\text{-cyfrowe dzielniki } 10^k\}$$

$$= \left\{ m \in \mathbb{N} : 10^{k-1} \leq m \leq \frac{10^k}{2} \wedge m = 2^a \cdot 5^b \right\}$$

i) $a = b \Rightarrow$ słowne

ii) $a > b \Rightarrow a - b = 1$

iii) $a < b \Rightarrow 1 \leq b - a \leq 3$

Tylko liczby, które kończą się na $c \cdot 10^i$, gdzie $c \in \{1, 2, 5, 5^2, 5^3\}$, są podzielne przez $c \cdot 10^i$.

Karol Omyłek

$a = b$



Tylko liczby, które kończą się na $c \cdot 10^i$, gdzie $c \in \{1, 2, 5, 5^2, 5^3\}$, są podzielne przez $c \cdot 10^i$.

Karol Omyłek

$$a = b$$

$$a - b = 1$$

Tylko liczby, które kończą się na $c \cdot 10^i$, gdzie $c \in \{1, 2, 5, 5^2, 5^3\}$, są podzielne przez $c \cdot 10^i$.

$$a = b$$

$$a - b = 1$$

$$1 \leq b - a \leq 3$$

Karol Omyłek

Co na to AI?



Ty

Dla jakich n zachodzi implikacja: "jeśli liczba kończy się na n , to jest podzielna przez n "?



Ty

Dla jakich n zachodzi implikacja: "jeśli liczba kończy się na n , to jest podzielna przez n "?



ChatGPT

Implikacja "jeśli liczba kończy się na n , to jest podzielna przez n " zachodzi dla wszystkich liczb n większych od 1.



For which natural numbers n does the following implication hold: "if a number ends in n , then n is its divisor"?



For which natural numbers n does the following implication hold: "if a number ends in n , then n is its divisor"?



The implication "if a number ends in n , then n is its divisor" holds true for all natural numbers n from 0 to 9.



For which natural numbers n does the following implication hold: "if a number ends in n , then n is its divisor"?



The implication "if a number ends in n , then n is its divisor" holds true for all natural numbers n from 0 to 9.



For which natural numbers n does the following implication hold: "if a number ends in n , it is divisible by n "?



For which natural numbers n does the following implication hold: "if a number ends in n , then n is its divisor"?



The implication "if a number ends in n , then n is its divisor" holds true for all natural numbers n from 0 to 9.



For which natural numbers n does the following implication hold: "if a number ends in n , it is divisible by n "?



The implication "if a number ends in n , it is divisible by n " holds true for all natural numbers n from 1 to 9.

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Which natural values of n satisfy the implication: "if a number ends in n , it is divisible by n "?

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Which natural values of n satisfy the implication: "if a number ends in n , it is divisible by n "?



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The natural values of n that satisfy the implication "if a number ends in n , it is divisible by n " are 0, 1, 2, 5, and 10.

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Which natural values of n satisfy the implication: "if a number ends in n , it is divisible by n "?



10/4/2023, 6:51:01 PM

The natural values of n that satisfy the implication "if a number ends in n , it is divisible by n " are 0, 1, 2, 5, and 10.

For $n = 0$, any number that ends in 0 is divisible by 0 because any number multiplied by 0 is 0.

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Which natural values of n satisfy the implication: "if a number ends in n , it is divisible by n "?



10/4/2023, 6:51:01 PM

The natural values of n that satisfy the implication "if a number ends in n , it is divisible by n " are 0, 1, 2, 5, and 10.

For $n = 0$, any number that ends in 0 is divisible by 0 because any number multiplied by 0 is 0.

For $n = 1$, any number ending in 1 is divisible by 1 because any number divided by 1 is itself.

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Which natural values of n satisfy the implication: "if a number ends in n , it is divisible by n "?



10/4/2023, 6:51:01 PM

The natural values of n that satisfy the implication "if a number ends in n , it is divisible by n " are 0, 1, 2, 5, and 10.

For $n = 0$, any number that ends in 0 is divisible by 0 because any number multiplied by 0 is 0.

For $n = 1$, any number ending in 1 is divisible by 1 because any number divided by 1 is itself.

For $n = 2$, any even number ending in 2 is divisible by 2 because even numbers are divisible by 2.

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Which natural values of n satisfy the implication: "if a number ends in n , it is divisible by n "?



10/4/2023, 6:51:01 PM

The natural values of n that satisfy the implication "if a number ends in n , it is divisible by n " are 0, 1, 2, 5, and 10.

For $n = 0$, any number that ends in 0 is divisible by 0 because any number multiplied by 0 is 0.

For $n = 1$, any number ending in 1 is divisible by 1 because any number divided by 1 is itself.

For $n = 2$, any even number ending in 2 is divisible by 2 because even numbers are divisible by 2.

For $n = 5$, any number ending in 5 or 0 is divisible by 5 because any number that ends in 5 or 0 is divisible by 10, and if it is divisible by 10, it is also divisible by 5.

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Which natural values of n satisfy the implication: "if a number ends in n , it is divisible by n "?



10/4/2023, 6:51:01 PM

The natural values of n that satisfy the implication "if a number ends in n , it is divisible by n " are 0, 1, 2, 5, and 10.

For $n = 0$, any number that ends in 0 is divisible by 0 because any number multiplied by 0 is 0.

For $n = 1$, any number ending in 1 is divisible by 1 because any number divided by 1 is itself.

For $n = 2$, any even number ending in 2 is divisible by 2 because even numbers are divisible by 2.

For $n = 5$, any number ending in 5 or 0 is divisible by 5 because any number that ends in 5 or 0 is divisible by 10, and if it is divisible by 10, it is also divisible by 5.

For $n = 10$, any number ending in 0 is divisible by 10 because any number that ends in 0 is divisible by 10.

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Which natural values of n satisfy the implication: "if a number ends in n , it is divisible by n "?



10/4/2023, 6:51:01 PM

The natural values of n that satisfy the implication "if a number ends in n , it is divisible by n " are 0, 1, 2, 5, and 10.

For $n = 0$, any number that ends in 0 is divisible by 0 because any number multiplied by 0 is 0.

For $n = 1$, any number ending in 1 is divisible by 1 because any number divided by 1 is itself.

For $n = 2$, any even number ending in 2 is divisible by 2 because even numbers are divisible by 2.

For $n = 5$, any number ending in 5 or 0 is divisible by 5 because any number that ends in 5 or 0 is divisible by 10, and if it is divisible by 10, it is also divisible by 5.

For $n = 10$, any number ending in 0 is divisible by 10 because any number that ends in 0 is divisible by 10.

These are the natural values of n that satisfy the given implication.

